

# Effects of Final-state Interactions on Mixing-induced $CP$ Violation in Penguin-dominated $B$ Decays

Hai-Yang Cheng,<sup>1</sup> Chun-Khiang Chua<sup>1</sup> and Amarjit Soni<sup>2</sup>

<sup>1</sup> Institute of Physics, Academia Sinica  
Taipei, Taiwan 115, Republic of China

<sup>2</sup> Physics Department, Brookhaven National Laboratory  
Upton, New York 11973

## Abstract

Motivated by the recent indications of the possibility of sizable deviations of the mixing-induced  $CP$  violation parameter,  $S_f$ , in the penguin-dominated  $b \rightarrow sq\bar{q}$  transition decays such as  $B^0 \rightarrow (\phi, \omega, \rho^0, \eta', \eta, \pi^0, f_0)K_S$  from  $\sin 2\beta$  determined from  $B \rightarrow J/\psi K_S$ , we study final-state rescattering effects on their decay rates and  $CP$  violation. Recent observations of large direct  $CP$  asymmetry in modes such as  $B^0 \rightarrow K^+\pi^-, \rho^-\pi^+$  means that final state phases in 2-body  $B$  decays may not be small. It is therefore important to examine these long-distance effects on penguin-dominated decays. Such long-distance effects on  $S_f$  are found to be generally small [i.e.  $\mathcal{O}(1 - 2\%)$ ] or negligible except for the  $\omega K_S$  and  $\rho^0 K_S$  modes where  $S_f$  is lowered by around 15% for the former and increased by the same percentage for the latter. However, final-state rescattering can enhance the  $\omega K_S$ ,  $\phi K_S$ ,  $\eta' K_S$ ,  $\rho^0 K_S$  and  $\pi^0 K_S$  rates significantly and flip the signs of direct  $CP$  asymmetries of the last two modes. Direct  $CP$  asymmetries in  $\omega K_S$  and  $\rho^0 K_S$  channels are predicted to be  $\mathcal{A}_{\omega K_S} \approx -0.13$  and  $\mathcal{A}_{\rho^0 K_S} \approx 0.47$ , respectively. However, direct  $CP$  asymmetry in all the other  $b \rightarrow s$  penguin dominated modes that we study is found to be rather small ( $\lesssim$  a few percents), rendering these modes a viable place to search for the  $CP$ -odd phases beyond the standard model. Since  $\Delta S_f (\equiv -\eta_f S_f - S_{J/\psi K_S})$ , with  $\eta_f$  being the  $CP$  eigenvalue of the final state  $f$ ) and  $\mathcal{A}_f$  are closely related, the theoretical uncertainties in the mixing induced parameter  $S_f$  and the direct  $CP$  asymmetry parameter  $\mathcal{A}_f$  are also coupled. Based on this work, it seems difficult to accommodate  $|\Delta S_f| > 0.10$  within the SM for  $B^0 \rightarrow (\phi, \omega, \rho^0, \eta', \eta, \pi^0)K_S$ , in particular,  $\eta' K_S$  is especially clean in our picture. For  $f_0 K_S$ , at present we cannot make reliable estimates. The sign of the central value of  $\Delta S_f$  for all the modes we study is positive but quite small, compared to the theoretical uncertainties, so that at present conclusive statements on the sign are difficult to make.

## I. INTRODUCTION AND MOTIVATION

Possible New Physics beyond the Standard Model is being intensively searched via the measurements of time-dependent  $CP$  asymmetries in neutral  $B$  meson decays into final  $CP$  eigenstates defined by

$$\frac{\Gamma(\overline{B}(t) \rightarrow f) - \Gamma(B(t) \rightarrow f)}{\Gamma(\overline{B}(t) \rightarrow f) + \Gamma(B(t) \rightarrow f)} = \mathcal{S}_f \sin(\Delta m t) + \mathcal{A}_f \cos(\Delta m t), \quad (1.1)$$

where  $\Delta m$  is the mass difference of the two neutral  $B$  eigenstates,  $\mathcal{S}_f$  monitors mixing-induced  $CP$  asymmetry and  $\mathcal{A}_f$  measures direct  $CP$  violation (in terms of the BaBar notation,  $\mathcal{C}_f = -\mathcal{A}_f$ ). The  $CP$ -violating parameters  $\mathcal{A}_f$  and  $\mathcal{S}_f$  can be expressed as

$$\mathcal{A}_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad \mathcal{S}_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad (1.2)$$

where

$$\lambda_f = \frac{q_B}{p_B} \frac{A(\overline{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)}. \quad (1.3)$$

In the standard model  $\lambda_f \approx \eta_f e^{-2i\beta}$  [see Eq. (2.14) below] for  $b \rightarrow s$  penguin-dominated or pure penguin modes with  $\eta_f = 1$  ( $-1$ ) for final  $CP$ -even (odd) states. Therefore, it is expected in the Standard Model that  $-\eta_f \mathcal{S}_f \approx \sin 2\beta$  and  $\mathcal{A}_f \approx 0$  with  $\beta$  being one of the angles of the unitarity triangle.

The mixing-induced  $CP$  violation in  $B$  decays has been already observed in the golden mode  $B^0 \rightarrow J/\psi K_S$  for several years. The current average of BaBar [1] and Belle [2] measurements is

$$\sin 2\beta \approx S_{J/\psi K_S} = 0.726 \pm 0.037. \quad (1.4)$$

However, the time-dependent  $CP$ -asymmetries in the  $b \rightarrow sq\bar{q}$  induced two-body decays such as  $B^0 \rightarrow (\phi, \omega, \pi^0, \eta', f_0) K_S$  are found to show some indications of deviations from the expectation of the Standard Model (SM). The BaBar [3] and Belle [4] results and their averages are shown in Table I. In the SM,  $CP$  asymmetry in all above-mentioned modes should be equal to  $S_{J/\psi K}$  with a small deviation *at most*  $\mathcal{O}(0.1)$  [5]. As discussed in [5], this may originate from the  $\mathcal{O}(\lambda^2)$  truncation and from the subdominant (color-suppressed) tree contribution to these processes. From Table I we see some possibly sizable deviations from the SM, especially in the  $\eta' K_S$  mode in which the discrepancy  $\Delta S_{\eta' K_S} = -0.30 \pm 0.11$  is a  $2.7\sigma$  effect where

$$\Delta S_f \equiv -\eta_f \mathcal{S}_f - S_{J/\psi K_S}. \quad (1.5)$$

If this deviation from  $S_{J/\psi K}$  is confirmed and established in the future, it may imply some New Physics beyond the SM.

In order to detect the signal of New Physics unambiguously in the penguin  $b \rightarrow s$  modes, it is of great importance to examine how much of the deviation of  $\mathcal{S}_f$  from  $S_{J/\psi K}$  is allowed in the SM [5, 6, 7, 8, 9, 10]. In all these previous studies and estimates of  $\Delta S_f$ , effects of FSI were not taken into account. In view of the striking observation of large direct  $CP$  violation in  $B^0 \rightarrow K^\pm \pi^\mp$ , it is clear that final-state phases in two-body  $B$  decays may not be small. It is therefore important to understand their effects on  $\Delta S_f$ .

TABLE I: Mixing-induced and direct  $CP$  asymmetries  $-\eta_f S_f$  (first entry) and  $\mathcal{A}_f$  (second entry), respectively, for various penguin-dominated modes with  $\eta_f$  being the  $CP$  eigenvalue of the final state. Experimental results are taken from [3, 4].

Final State	BaBar [3]	Belle [4]	Average
$\phi K_S$	$0.50 \pm 0.25^{+0.07}_{-0.04}$	$0.08 \pm 0.33 \pm 0.09$	$0.35 \pm 0.20$
	$-0.00 \pm 0.23 \pm 0.05$	$0.08 \pm 0.22 \pm 0.09$	$0.04 \pm 0.17$
$\omega K_S$	$0.50^{+0.34}_{-0.38} \pm 0.02$	$0.76 \pm 0.65^{+0.13}_{-0.16}$	$0.55^{+0.30}_{-0.32}$
	$0.56^{+0.29}_{-0.27} \pm 0.03$	$0.27 \pm 0.48 \pm 0.15$	$0.48 \pm 0.25$
$\eta' K_S$	$0.30 \pm 0.14 \pm 0.02$	$0.65 \pm 0.18 \pm 0.04$	$0.43 \pm 0.11$
	$0.21 \pm 0.10 \pm 0.02$	$-0.19 \pm 0.11 \pm 0.05$	$0.04 \pm 0.08$
$\pi^0 K_S$	$0.35^{+0.30}_{-0.33} \pm 0.04$	$0.32 \pm 0.61 \pm 0.13$	$0.34^{+0.27}_{-0.29}$
	$-0.06 \pm 0.18 \pm 0.03$	$-0.11 \pm 0.20 \pm 0.09$	$-0.08 \pm 0.14$
$f_0 K_S$	$0.95^{+0.23}_{-0.32} \pm 0.10$	$-0.47 \pm 0.41 \pm 0.08$	$0.39 \pm 0.26$
	$0.24 \pm 0.31 \pm 0.15$	$-0.39 \pm 0.27 \pm 0.09$	$-0.14 \pm 0.22$

The decay amplitude for the pure penguin or penguin-dominated charmless  $B$  decay in general has the form

$$M(\overline{B}^0 \rightarrow f) = V_{ub}V_{us}^* F^u + V_{cb}V_{cs}^* F^c + V_{tb}V_{ts}^* F^t. \quad (1.6)$$

Unitarity of the CKM matrix elements leads to

$$M(\overline{B}^0 \rightarrow f) = V_{ub}V_{us}^* A_f^u + V_{cb}V_{cs}^* A_f^c \approx \lambda \lambda^4 R_b e^{-i\gamma} A_f^u + \lambda \lambda^2 A_f^c, \quad (1.7)$$

where  $A_f^u = F^u - F^t$ ,  $A_f^c = F^c - F^t$ ,  $R_b \equiv |V_{ud}V_{ub}/(V_{cd}V_{cb})| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$  with  $\bar{\rho}, \bar{\eta}, A, \lambda$  being the Wolfenstein parameters [11]. The first term is suppressed by a factor of  $\lambda^2$  relative to the second term. For a pure penguin decay such as  $B^0 \rightarrow \phi K^0$ , it is naively expected that  $A_f^u$  is in general comparable to  $A_f^c$  in magnitude. Therefore, to a good approximation  $-\eta_f S_f \approx \sin 2\beta \approx S_{J/\psi K}$ . For penguin-dominated modes such as  $\omega K_S, \rho^0 K_S, \pi^0 K_S$ ,  $A_f^u$  also receives tree contributions from the  $b \rightarrow u\bar{u}s$  tree operators. Since the Wilson coefficient for the penguin operator is smaller than the one for the tree operator,  $A_f^u$  could be significantly larger than  $A_f^c$ . As the first term carries a weak phase  $\gamma$ , it is possible that  $S_f$  is subject to a significant “tree pollution”. To quantify the deviation, it is known that to the first order in  $r_f \equiv (\lambda_u A_f^u)/(\lambda_c A_f^c)$  [8, 12]

$$\Delta S_f = 2|r_f| \cos 2\beta \sin \gamma \cos \delta_f, \quad \mathcal{A}_f = 2|r_f| \sin \gamma \sin \delta_f, \quad (1.8)$$

with  $\delta_f = \arg(A_f^u/A_f^c)$ . Hence, the magnitude of the  $CP$  asymmetry difference  $\Delta S_f$  and direct  $CP$  violation are both governed by the size of  $A_f^u/A_f^c$ . However, for the aforementioned penguin-dominated modes, the tree contribution is color suppressed and hence in practice the deviation of  $S_f$  is expected to be small [5]. (However, we shall see below that a sizable  $\Delta S_f$  can occur in  $\omega K_S$  and  $\rho^0 K_S$  modes. For a review of model calculations of  $\Delta S_f$ , see [13].)

Since the penguin loop contributions are sensitive to high virtuality, New Physics beyond the SM may contribute to  $S_f$  through the heavy particles in the loops (for a review of the New Physics

sources contributing to  $S_f$ , see [14]). Another possibility is that final-state interactions are the possible tree pollution sources to  $S_f$ . Both  $A_f^u$  and  $A_f^c$  will receive long-distance tree and penguin contributions from rescattering of some intermediate states. In particular there may be some dynamical enhancement of light  $u$ -quark loop. If tree contributions to  $A_f^u$  are sizable, then final-state rescattering will have the potential of pushing  $S_f$  away from the naive expectation. Take the penguin-dominated decay  $\overline{B}^0 \rightarrow \omega \overline{K}^0$  as an illustration. It can proceed through the weak decay  $\overline{B}^0 \rightarrow K^{*-} \pi^+$  followed by the rescattering  $K^{*-} \pi^+ \rightarrow \omega \overline{K}^0$ . The tree contribution to  $\overline{B}^0 \rightarrow K^{*-} \pi^+$ , which is color allowed, turns out to be comparable to the penguin one because of the absence of the chiral enhancement characterized by the  $a_6$  penguin term. Consequently, even within the framework of the SM, final-state rescattering may provide a mechanism of tree pollution to  $S_f$ . By the same token, we note that although  $\overline{B}^0 \rightarrow \phi \overline{K}^0$  is a pure penguin process at short distances, it does receive tree contributions via long-distance rescattering.

In this work, we shall study the effects of final-state interactions (FSI) on the time-dependent  $CP$  asymmetries  $S_f$  and  $\mathcal{A}_f$ . In [15] we have studied the final-state rescattering effects on the hadronic  $B$  decays and examined their impact on direct  $CP$  violation. The direct  $CP$ -violating partial rate asymmetries in charmless  $B$  decays to  $\pi\pi/\pi K$  and  $\rho\pi$  are significantly affected by final-state rescattering and their signs are generally different from that predicted by the short-distance approach such as QCD factorization [16, 18, 19]. Evidence of direct  $CP$  violation in the decay  $\overline{B}^0 \rightarrow K^- \pi^+$  is now established, while the combined BaBar and Belle measurements of  $\overline{B}^0 \rightarrow \rho^\pm \pi^\mp$  imply a  $3.6\sigma$  direct  $CP$  asymmetry in the  $\rho^+ \pi^-$  mode [17]. In fact, direct  $CP$  asymmetries in these channels are a lot bigger than expectations (of many people) and may be indicative of appreciable LD rescattering effects, in general, in  $B$  decays. Our predictions for  $CP$  violation agree with experiment in both magnitude and sign, whereas the QCD factorization predictions (especially for  $\rho^+ \pi^-$ ) [19] seem to have some difficulties with the data.

Besides some significant final-state rescattering effects on direct  $CP$  violation, another motivation for including FSIs is that there are consistently 2 to 3  $\sigma$  deviations between the central values of the QCDF predictions for penguin-dominated modes such as  $B \rightarrow K^* \pi$ ,  $K \phi$ ,  $K^* \phi$ ,  $K \eta'$  and the experimental data [19]. This discrepancy between theory and experiment for branching ratios may indicate the importance of subleading power corrections such as FSI effects and/or the annihilation topology.

Since direct  $CP$  violation in charmless  $B$  decays can be significantly affected by final-state rescattering, it is clearly important to try to take into account the FSI effect on the mixing-induced and direct  $CP$  asymmetries  $S_f$  and  $\mathcal{A}_f$  of these penguin dominated modes. The layout of the present paper is as follows. In Sec. II we discuss the short-distance contributions to the  $b \rightarrow sq\bar{q}$  transition decays  $B^0 \rightarrow (\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0) K_S$  within the framework of QCD factorization. We then proceed to study the final-state rescattering effects on  $CP$  asymmetries  $S_f$  and  $\mathcal{A}_f$  in Sec. III. Sec. IV contains our conclusions.

## II. SHORT-DISTANCE CONTRIBUTIONS

### A. Decay amplitudes in QCD factorization

We shall use the QCD factorization approach [16, 18, 19] to study the short-distance contributions to the decays  $\overline{B}^0 \rightarrow (\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0) \overline{K}^0$ . In QCD factorization, the factorization amplitudes of above-mentioned decays are given by

$$\begin{aligned}
\langle \phi \overline{K}^0 | H_{\text{eff}} | \overline{B}^0 \rangle &= \frac{G_F}{\sqrt{2}} (p_B \cdot \varepsilon_\phi^*) \sum_{p=u,c} \lambda_p \left\{ \left[ (a_3 + a_5) (\overline{K}^0 \phi) + (a_4^p + r_\chi^\phi a_6^p) (\overline{K}^0 \phi) - \frac{1}{2} (a_7 + a_9) (\overline{K}^0 \phi) \right] \right. \\
&\quad \times \left. 2f_\phi m_\phi F_1^{BK}(m_\phi^2) + f_B f_K f_\phi \left[ b_3 (\overline{K}^0 \phi) - \frac{1}{2} b_3^{\text{EW}} (\overline{K}^0 \phi) \right] \frac{2m_\phi}{m_B^2} \right\}, \\
\langle \omega \overline{K}^0 | H_{\text{eff}} | \overline{B}^0 \rangle &= \frac{G_F}{2} (p_B \cdot \varepsilon_\omega^*) \sum_{p=u,c} \lambda_p \left\{ \left[ a_2 (\overline{K}^0 \omega) \delta_u^p + 2(a_3 + a_5) (\overline{K}^0 \omega) + \frac{1}{2} (a_7 + a_9) (\overline{K}^0 \omega) \right] \right. \\
&\quad \times \left. 2f_\omega m_\omega F_1^{BK}(m_\omega^2) + \left[ (a_4^p - \mu_\chi^K a_6^p) (\omega \overline{K}^0) - \frac{1}{2} (a_{10}^p - \mu_\chi^K a_8^p) (\omega \overline{K}^0) \right] \right. \\
&\quad \times \left. 2f_K m_\omega A_0^{B\omega}(m_K^2) + f_B f_K f_\omega \left[ b_3 (\omega \overline{K}^0) - \frac{1}{2} b_3^{\text{EW}} (\omega \overline{K}^0) \right] \frac{2m_\omega}{m_B^2} \right\}, \\
\langle \rho^0 \overline{K}^0 | H_{\text{eff}} | \overline{B}^0 \rangle &= \frac{G_F}{2} (p_B \cdot \varepsilon_\rho^*) \sum_{p=u,c} \lambda_p \left\{ \left[ a_2 (\overline{K}^0 \rho) \delta_u^p + \frac{3}{2} (a_7 + a_9) (\overline{K}^0 \rho) \right] \right. \\
&\quad \times \left. 2f_\rho m_\rho F_1^{BK}(m_\rho^2) - \left[ (a_4^p - \mu_\chi^K a_6^p) (\rho \overline{K}^0) - \frac{1}{2} (a_{10}^p - \mu_\chi^K a_8^p) (\rho \overline{K}^0) \right] \right. \\
&\quad \times \left. 2f_K m_\rho A_0^{B\rho}(m_K^2) - f_B f_K f_\rho \left[ b_3 (\rho \overline{K}^0) - \frac{1}{2} b_3^{\text{EW}} (\rho \overline{K}^0) \right] \frac{2m_\rho}{m_B^2} \right\}, \\
\langle \pi^0 \overline{K}^0 | H_{\text{eff}} | \overline{B}^0 \rangle &= i \frac{G_F}{2} \sum_{p=u,c} \lambda_p \left\{ \left[ a_2 (\overline{K}^0 \pi^0) \delta_u^p + \frac{3}{2} \alpha_{3,\text{EW}} (\overline{K}^0 \pi^0) \right] f_\pi F_0^{BK}(m_\pi^2) (m_B^2 - m_K^2) \right. \\
&\quad - \left[ \alpha_4^p (\pi^0 \overline{K}^0) - \frac{1}{2} \alpha_{4,\text{EW}}^p (\pi^0 \overline{K}^0) \right] f_K F_0^{B\pi}(m_K^2) (m_B^2 - m_\pi^2) \\
&\quad - \left. f_B f_K f_\pi \left[ b_3 (\pi^0 \overline{K}^0) - \frac{1}{2} b_3^{\text{EW}} (\pi^0 \overline{K}^0) \right] \right\}, \\
\langle \eta' \overline{K}^0 | H_{\text{eff}} | \overline{B}^0 \rangle &= i \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left\{ F_0^{BK}(m_{\eta'}^2) (m_B^2 - m_K^2) \left\{ \frac{f_{\eta'}^q}{\sqrt{2}} \left[ a_2 (\overline{K}^0 \eta'_q) \delta_u^p + 2\alpha_3 (\overline{K}^0 \eta'_q) \right. \right. \right. \\
&\quad + \left. \left. \frac{1}{2} \alpha_{3,\text{EW}} (\overline{K}^0 \eta'_q) \right] + f_{\eta'}^c \left[ a_2 (\overline{K}^0 \eta'_c) \delta_c^p + \alpha_3 (\overline{K}^0 \eta'_c) \right] \right. \\
&\quad + \left. \left. f_{\eta'}^s \left[ \alpha_3 (\overline{K}^0 \eta'_s) + \alpha_4^p (\overline{K}^0 \eta'_s) - \frac{1}{2} \alpha_{3,\text{EW}} (\overline{K}^0 \eta'_s) - \frac{1}{2} \alpha_{4,\text{EW}}^p (\eta'_s \overline{K}^0) \right] \right\} \right. \\
&\quad + \left. F_0^{B\eta'}(m_K^2) (m_B^2 - m_{\eta'}^2) \frac{f_K}{\sqrt{2}} \left[ \alpha_4^p (\eta' \overline{K}^0) - \frac{1}{2} \alpha_{4,\text{EW}}^p (\eta' \overline{K}^0) \right] \right. \\
&\quad + \left. f_B f_K \left( \frac{1}{\sqrt{2}} f_{\eta'}^q + f_{\eta'}^s \right) \left[ b_3 (\eta' \overline{K}^0) - \frac{1}{2} b_3^{\text{EW}} (\eta' \overline{K}^0) \right] \right\},
\end{aligned}$$

$$\begin{aligned}
\langle f_0 \bar{K}^0 | H_{\text{eff}} | \bar{B}^0 \rangle = & -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left\{ \left[ (a_4^p - r_\chi^K a_6^p)(f_0 \bar{K}^0) - \frac{1}{2}(a_{10}^p - r_\chi^K a_8^p)(f_0 \bar{K}^0) \right] \right. \\
& \times f_K F_0^{Bf_0}(m_K^2)(m_B^2 - m_{f_0}^2) + (2a_6^p - a_8^p)(\bar{K}^0 f_0) \bar{f}_s \frac{m_{f_0}}{m_b} F_0^{BK}(m_{f_0}^2)(m_B^2 - m_K^2) \\
& \left. - f_B f_K (\bar{f}_s + \bar{f}_u) \left[ b_3(f_0 \bar{K}^0) - \frac{1}{2} b_3^{\text{EW}}(f_0 \bar{K}^0) \right] \right\}, \quad (2.1)
\end{aligned}$$

where  $F_{1,0}$  and  $A_0$  denote pseudoscalar and vector form factors in the standard convention [20],  $\lambda_p \equiv V_{pb} V_{ps}^*$ ,

$$\begin{aligned}
\alpha_3(M_1 M_2) &= a_3(M_1 M_2) - a_5(M_1 M_2), & \alpha_4^p(M_1 M_2) &= a_4^p(M_1 M_2) + r_\chi^{M_2} a_6^p(M_1 M_2), \\
\alpha_{3,\text{EW}}(M_1 M_2) &= -a_7(M_1 M_2) + a_9(M_1 M_2), & \alpha_{4,\text{EW}}^p(M_1 M_2) &= a_{10}^p(M_1 M_2) + r_\chi^{M_2} a_8^p(M_1 M_2),
\end{aligned} \quad (2.2)$$

and

$$\begin{aligned}
r_\chi^\phi(\mu) &= \frac{2m_\phi}{m_b(\mu)} \frac{f_\phi^\perp(\mu)}{f_\phi}, & r_\chi^\pi(\mu) &= \frac{2m_\pi^2}{2m_b(\mu)m_q(\mu)}, \\
r_\chi^K(\mu) &= \frac{2m_K^2}{m_b(\mu)[m_s(\mu) + m_q(\mu)]}, & r_\chi^{\eta'}(\mu) &= \frac{2m_{\eta'}^2}{2m_b(\mu)m_s(\mu)} \left( 1 - \frac{f_{\eta'}^q}{\sqrt{2}f_{\eta'}^s} \right), \quad (2.3)
\end{aligned}$$

with the scale dependent transverse decay constant  $f_V^\perp$  being defined as

$$\langle V(p, \varepsilon^*) | \bar{q} \sigma_{\mu\nu} q' | 0 \rangle = f_V^\perp (p_\mu \varepsilon_\nu^* - p_\nu \varepsilon_\mu^*). \quad (2.4)$$

Note that the  $a_6$  penguin term appears in the decay amplitude of  $\bar{B}^0 \rightarrow \phi \bar{K}^0$  owing to the non-vanishing transverse decay constant of the  $\phi$  meson. The scalar decay constant  $\bar{f}_q$  of  $f_0(980)$  in Eq. (2.1) is defined by  $\langle f_0 | \bar{q} q | 0 \rangle = m_{f_0} \bar{f}_q$ . The decay amplitude for  $\bar{B}^0 \rightarrow \eta \bar{K}^0$  is obtained from  $\bar{B}^0 \rightarrow \eta' \bar{K}^0$  by replacing  $\eta' \rightarrow \eta$ . Note that the use of nonzero  $q^2$  in the argument of form factors in Eq. (2.1) means that some corrections quadratic in the light quark masses are automatically incorporated.

The effective parameters  $a_i^p$  with  $p = u, c$  can be calculated in the QCD factorization approach [16]. They are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [16, 19]

$$a_i^p(M_1 M_2) = c_i + \frac{c_{i\pm 1}}{N_c} + \frac{c_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] + P_i^p(M_2), \quad (2.5)$$

where  $i = 1, \dots, 10$ , the upper (lower) signs apply when  $i$  is odd (even),  $c_i$  are the Wilson coefficients,  $C_F = (N_c^2 - 1)/(2N_c)$  with  $N_c = 3$ ,  $M_2$  is the emitted meson and  $M_1$  shares the same spectator quark with the  $B$  meson. The quantities  $V_i(M_2)$  account for vertex corrections,  $H_i(M_1 M_2)$  for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the  $B$  meson and  $P_i(M_2)$  for penguin contractions. The explicit expressions of these quantities together with the annihilation quantities  $b_3$  and  $b_3^{\text{EW}}$  can be found in [16, 19].

For  $B \rightarrow \eta^{(\prime)} K$  decay,  $\eta_q^{(\prime)}$  and  $\eta_s^{(\prime)}$  in Eq. (2.1) refer to the non-strange and strange quark states, respectively, of  $\eta^{(\prime)}$ . The decay constants  $f_{\eta^{(\prime)}}^{q,s,c}$  are defined by

$$\begin{aligned}
\langle \eta^{(\prime)}(p) | \bar{q} \gamma_\mu \gamma_5 q | 0 \rangle &= -\frac{i}{\sqrt{2}} f_{\eta^{(\prime)}}^q p_\mu, & \langle \eta^{(\prime)}(p) | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle &= -i f_{\eta^{(\prime)}}^s p_\mu, & \langle \eta^{(\prime)}(p) | \bar{c} \gamma_\mu \gamma_5 c | 0 \rangle &= -i f_{\eta^{(\prime)}}^c p_\mu,
\end{aligned} \quad (2.6)$$

with  $q = u$  or  $d$ . Numerically, we shall use [21]

$$\begin{aligned} f_{\eta'}^q &= 89 \text{ MeV}, & f_{\eta'}^s &= 131 \text{ MeV}, & f_{\eta'}^c &= -6.3 \text{ MeV}, \\ f_{\eta}^q &= 108 \text{ MeV}, & f_{\eta}^s &= -111 \text{ MeV}, & f_{\eta}^c &= -2.4 \text{ MeV}, \end{aligned} \quad (2.7)$$

recalling that, in our convention,  $f_{\pi} = 132 \text{ MeV}$ . As for the  $B \rightarrow \eta^{(\prime)}$  form factor, we shall follow [18] to use the relation  $F_0^{B\eta^{(\prime)}} = (f_{\eta^{(\prime)}}^q/f_{\eta^{(\prime)}}^s)F_0^{B\pi}$  to obtain the values of the form factors  $F_0^{B\eta^{(\prime)}}$ . The wave functions of the physical  $\eta'$  and  $\eta$  states are related to that of the SU(3) singlet state  $\eta_0$  and octet state  $\eta_8$  by

$$\eta' = \eta_8 \sin \phi + \eta_0 \cos \phi, \quad \eta = \eta_8 \cos \phi - \eta_0 \sin \phi, \quad (2.8)$$

with the mixing angle  $\phi = -(15.4 \pm 1.0)^\circ$  [21].

The decay  $B \rightarrow f_0(980)K$  has been discussed in detail in [22]. Since the scalar meson  $f_0(980)$  cannot be produced via the vector current owing to charge conjugation invariance or conservation of vector current, the tree contribution to  $\bar{B}^0 \rightarrow f_0 \bar{K}^0$  vanishes under the factorization approximation. Just as the SU(3)-singlet  $\eta_0$ ,  $f_0(980)$  in the 2-quark picture also contains strange and non-strange quark content

$$|f_0(980)\rangle = |\bar{s}s\rangle \cos \theta + |\bar{n}n\rangle \sin \theta, \quad (2.9)$$

with  $\bar{n}n \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ . Experimental implications for the mixing angle  $\theta$  have been discussed in detail in [22, 23]; it lies in the ranges of  $25^\circ < \theta < 40^\circ$  and  $140^\circ < \theta < 165^\circ$ . Based on the QCD sum-rule technique, the decay constants  $\tilde{f}_s$  and  $\tilde{f}_n$  defined by  $\langle f_0^q | \bar{q}q | 0 \rangle = m_{f_0} \tilde{f}_q$  with  $f_0^n = \bar{n}n$  and  $f_0^s = \bar{s}s$  have been estimated in [22] by taking into account their scale dependence and radiative corrections. It turns out that  $\tilde{f}_s(1 \text{ GeV}) \approx 0.33 \text{ GeV}$  [22], for example. In the two-quark scenario for  $f_0(980)$ , the decay constants  $\bar{f}_{n,s}$  are related to  $\tilde{f}_{n,s}$  via [22]

$$\bar{f}_s = \tilde{f}_s \cos \theta, \quad \bar{f}_n = \tilde{f}_n \sin \theta. \quad (2.10)$$

The hard spectator function relevant for  $B \rightarrow f_0 K$  decay has the form

$$H_i(f_0 K) = \frac{\bar{f}_u f_B}{F_0^B f_0^u(0) m_B^2} \int_0^1 \frac{d\rho}{\rho} \Phi_B(\rho) \int_0^1 \frac{d\xi}{\xi} \Phi_K(\xi) \int_0^1 \frac{d\eta}{\bar{\eta}} \left[ \Phi_{f_0}(\eta) + \frac{2m_{f_0}}{m_b} \frac{\bar{\xi}}{\xi} \Phi_{f_0}^p(\eta) \right], \quad (2.11)$$

for  $i = 1, 4, 10$  and  $H_i = 0$  for  $i = 6, 8$ . where  $\bar{\xi} \equiv 1 - \xi$  and  $\bar{\eta} = 1 - \eta$ . As for the parameters  $a_{6,8}^{u,c}(\bar{K}^0 f_0)$  appearing in Eq. (2.1), they have the same expressions as  $a_{6,8}^{u,c}(f_0 \bar{K}^0)$  except that the penguin function  $\hat{G}_K$  (see Eq. (55) of the second reference in [16]) is replaced by  $\hat{G}_{f_0}$  and  $\Phi_K^p$  by  $\Phi_{f_0}^p$ . For the distribution amplitudes  $\Phi_{f_0}$ ,  $\Phi_{f_0}^p$  and the annihilation amplitudes, see [22] for details.

Although the parameters  $a_i (i \neq 6, 8)$  and  $a_{6,8} r_\chi$  are formally renormalization scale and  $\gamma_5$  scheme independent, in practice there exists some residual scale dependence in  $a_i(\mu)$  to finite order. To be specific, we shall evaluate the vertex corrections to the decay amplitude at the scale  $\mu = m_b$ . In contrast, as stressed in [16], the hard spectator and annihilation contributions should be evaluated at the hard-collinear scale  $\mu_h = \sqrt{\mu \Lambda_h}$  with  $\Lambda_h \approx 500 \text{ MeV}$ . There is one more serious complication about these contributions; that is, while QCD factorization predictions are model independent in the  $m_b \rightarrow \infty$  limit, power corrections always involve troublesome endpoint divergences. For

example, the annihilation amplitude has endpoint divergences even at twist-2 level and the hard spectator scattering diagram at twist-3 order is power suppressed and possesses soft and collinear divergences arising from the soft spectator quark. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. We shall follow [16] to parametrize the endpoint divergence  $X_A \equiv \int_0^1 dx/(1-x)$  in the annihilation diagram as

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right) (1 + \rho_A e^{i\phi_A}) \quad (2.12)$$

with  $\rho_A \leq 1$ . Likewise, the endpoint divergence  $X_H$  in the hard spectator contributions can be parametrized in a similar way.

## B. Consideration of mixing-induced $CP$ asymmetry

Consider the mixing-induced  $CP$  violation in the decay modes  $(\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0)K_S$  mediated by  $b \rightarrow sq\bar{q}$  transitions. Since a common final state is reached only via  $K^0 - \bar{K}^0$  mixing, hence

$$\begin{aligned} \lambda_f &= \frac{q_B}{p_B} \frac{q_K}{p_K} \frac{A(\bar{B}^0 \rightarrow M\bar{K}^0)}{A(B^0 \rightarrow MK^0)} \\ &\approx \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \frac{A(\bar{B}^0 \rightarrow M\bar{K}^0)}{A(B^0 \rightarrow MK^0)}. \end{aligned} \quad (2.13)$$

We shall use this expression for  $\lambda_f$  to compute  $CP$  asymmetries  $S_f$  and  $\mathcal{A}_f$ . For  $M = V$ ,  $A(B^0 \rightarrow VK^0)$  has the same expression as  $-A(\bar{B}^0 \rightarrow V\bar{K}^0)$  with the CKM mixing angles  $\lambda_p \rightarrow \lambda_p^*$  owing to  $CP|VK^0\rangle = -|V\bar{K}^0\rangle$ , while for  $M = P$ ,  $A(B^0 \rightarrow PK^0)$  is obtained from  $A(\bar{B}^0 \rightarrow P\bar{K}^0)$  with  $\lambda_p \rightarrow \lambda_p^*$ . If the contributions from  $V_{ub}V_{us}^*$  terms are neglected, then we will have

$$\frac{A(\bar{B}^0 \rightarrow M\bar{K}^0)}{A(B^0 \rightarrow MK^0)} \approx \eta_f \frac{V_{cd}V_{cs}^*}{V_{cd}^*V_{cs}} \Rightarrow \lambda_f \approx \eta_f \frac{V_{td}}{V_{td}^*} = \eta_f e^{-2i\beta} \Rightarrow -\eta_f S_f \approx \sin 2\beta. \quad (2.14)$$

Note that  $\eta_f = 1$  for  $f_0 K_S$  and  $\eta_f = -1$  for  $(\phi, \omega, \rho^0, \eta', \eta, \pi^0) K_S$ .

From Eq. (2.1) we see that among the seven modes under consideration, only  $\omega K_S$ ,  $\rho^0 K_S$ ,  $\pi^0 K_S$  and  $\eta^{(\prime)} K_S$  receive tree contributions from the tree diagram  $b \rightarrow sq\bar{q}$  ( $q = u, d$ ). However, since the tree contribution is color suppressed, the deviation of  $-\eta_f S_f$  from  $\sin 2\beta$  is expected to be small. Nevertheless, the large cancellation between  $a_4$  and  $a_6$  penguin terms in the amplitudes of  $\bar{B}^0 \rightarrow \omega\bar{K}^0$  and  $\bar{B}^0 \rightarrow \rho^0\bar{K}^0$  render the tree contribution relatively significant. Hence,  $\Delta S_f$  is expected to be largest in the  $\omega K_S$  and  $\rho^0 K_S$  modes. Since the typical values of the effective Wilson parameters obtained from Eq. (2.5) are

$$\begin{aligned} a_2 &\approx 0.18 - 0.11i, & a_3 &\approx -0.003 + 0.005i, & a_5 &\approx 0.008 - 0.006i, \\ a_4^u &\approx -0.03 - 0.02i, & a_4^c &\approx -0.03 - 0.006i, \\ a_6^u &\approx -0.06 - 0.02i, & a_6^c &\approx -0.06 - 0.004i, \end{aligned} \quad (2.15)$$

and  $r_\chi^K \approx 0.57$ , it is not difficult to see from Eq. (2.1) that  $\delta_f$  lies in the region  $0 > \delta_f > -\pi/2$  for the  $\omega K_S$  and  $f_0 K_S$  modes,  $\pi > \delta_f > \pi/2$  for  $\rho^0 K_S$  and  $\pi/2 > \delta_f > 0$  for the remaining three ones. Therefore, based purely on SD contributions, it is expected that  $\Delta S_f > 0$  for all the modes except for  $\rho^0 K_S$  and that  $\mathcal{A}_f$  is negative for  $\omega K_S, f_0 K_S$  and positive for  $\phi K_S, \rho^0 K_S, \pi^0 K_S, \eta^{(\prime)} K_S$ .



### C. Numerical results

To proceed with the numerical calculations, we shall follow [16, 19] for the choices of the relevant parameters except for the form factors and CKM matrix elements. For form factors we shall use those derived in the covariant light-front quark model [24] and assign a common value of 0.03 for the form factor errors, e.g.  $F^{B\pi}(0) = 0.25 \pm 0.03$ . For CKM matrix elements, see the unitarity triangle analysis in [25]. For definiteness, we shall follow the first reference in [25] to use the Wolfenstein parameters  $A = 0.801$ ,  $\lambda = 0.2265$ ,  $\bar{\rho} = 0.189$  and  $\bar{\eta} = 0.358$  which correspond to  $\sin 2\beta = 0.723$  and  $\gamma = 63^\circ$ . We assign  $15^\circ$  error for the unitarity angle  $\gamma$ , recalling that two values  $\gamma = (62_{-12}^{+10})^\circ$  and  $\gamma = (64 \pm 18)^\circ$  are obtained in [25]. For endpoint divergences encountered in hard spectator and annihilation contributions we take the default values  $\rho_A = \rho_H = 0$ . We will return back to this point below when discussing long-distance rescattering effects.

The obtained branching ratios for the decays  $\bar{B}^0 \rightarrow (\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0) \bar{K}^0$  are shown in the second column of Table II, while the corresponding  $CP$  violation asymmetries  $S_f$  and  $\mathcal{A}_f$  are depicted in Table III. In general, our results for branching ratios and direct  $CP$  asymmetries are in agreement with [19]. Some differences result from different inputs of the form factors and CKM parameters. It is evident that, as far as the central values are concerned, the predicted branching ratios by the short-distance (SD) QCD factorization approach are generally too low compared to experiment especially for  $\omega \bar{K}^0$ ,  $\rho^0 K_S$  and  $\pi^0 \bar{K}^0$ . Note that  $\mathcal{B}(\bar{B}^0 \rightarrow \omega \bar{K}^0) \lesssim 10^{-6}$  is predicted in the early QCD factorization calculation [26]. The very large (small) branching ratio for  $\eta' \bar{K}^0$  ( $\eta \bar{K}^0$ ) is understandable as follows. There are two distinct penguin contributions to  $\eta^{(\prime)} \bar{K}^0$ : one couples to the  $d$  quark content of the  $\eta^{(\prime)}$ , while the other is related to the  $s$  quark component of the  $\eta^{(\prime)}$  [see also Eq. (2.1)]. If the  $\eta - \eta'$  mixing angle is given by  $-19.5^\circ$ , the expressions of the  $\eta'$  and  $\eta$  wave functions will become very simple:

$$|\eta'\rangle = \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d + 2\bar{s}s\rangle, \quad |\eta\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d - \bar{s}s\rangle. \quad (2.16)$$

It is evident that the SD  $\eta \bar{K}^0$  amplitude vanishes in SU(3) limit, whereas the constructive interference between the penguin amplitudes accounts for the large rate of  $\eta' \bar{K}^0$ . In reality the  $\eta - \eta'$  mixing angle is  $-(15.4 \pm 1.0)^\circ$  [21], but this does not affect the above physical picture.

Owing to the large cancellation between the  $a_4$  and  $a_6$  penguin terms, the main contribution to the decay  $\bar{B}^0 \rightarrow f_0 \bar{K}^0$  arises from the penguin diagram involving the strange quark content of  $f_0(980)$ , namely, the term with the scalar decay constant  $\bar{f}_s$ . Consequently, the maximal branching ratio  $9.9 \times 10^{-6}$  occurs near the zero mixing angle. The result of  $\mathcal{B}(\bar{B}^0 \rightarrow f_0 \bar{K}^0) = 8.1 \times 10^{-6}$  shown in Table II corresponds to  $\theta = 150^\circ$ . Note that the decay  $\bar{B}^0 \rightarrow f_0(980) \bar{K}^0$  was measured by BaBar [27] with the result

$$\mathcal{B}(B^0 \rightarrow f_0(980) K^0 \rightarrow \pi^+ \pi^- K^0) = (6.0 \pm 0.9 \pm 1.3) \times 10^{-6}. \quad (2.17)$$

The absolute branching ratios for  $B \rightarrow f_0 K$  depends critically on the branching fraction of  $f_0(980) \rightarrow \pi\pi$ . We use the results from the most recent analysis of [28] to obtain  $\mathcal{B}(f_0(980) \rightarrow \pi\pi) = 0.80 \pm 0.14$  and  $\mathcal{B}(B^0 \rightarrow f_0(980) K^0) = (11.3 \pm 3.6) \times 10^{-6}$  as shown in Table II.<sup>1</sup> In short, al-

<sup>1</sup> For comparison, the world average of the branching ratio for  $B^- \rightarrow f_0 K^- \rightarrow \pi^+ \pi^- K^-$  is  $(8.49_{-1.26}^{+1.35}) \times 10^{-6}$

TABLE II: Short-distance (SD) and long-distance (LD) contributions to the branching ratios (in units of  $10^{-6}$ ) for various penguin-dominated modes. The first and second theoretical errors correspond to the SD and LD ones, respectively (see the text for details). The world averages of experimental measurements are taken from [17].

	SD	SD+LD	Expt
$\overline{B}^0 \rightarrow \phi \overline{K}^0$	$5.6^{+1.9}_{-1.8}$	$8.6^{+1.2+2.9}_{-1.2-1.8}$	$8.3^{+1.2}_{-1.0}$
$\overline{B}^0 \rightarrow \omega \overline{K}^0$	$2.0^{+3.5}_{-1.3}$	$5.6^{+2.9+3.7}_{-1.2-2.1}$	$5.6 \pm 0.9$
$\overline{B}^0 \rightarrow \rho^0 \overline{K}^0$	$2.8^{+3.2}_{-1.6}$	$5.2^{+3.2+2.6}_{-1.5-1.2}$	$5.1 \pm 1.6$
$\overline{B}^0 \rightarrow \eta' \overline{K}^0$	$42.1^{+45.6}_{-19.4}$	$69.4^{+51.3+50.4}_{-21.4-19.2}$	$68.6 \pm 4.2$
$\overline{B}^0 \rightarrow \eta \overline{K}^0$	$1.8^{+1.2}_{-0.9}$	$1.8^{+1.2+0.1}_{-0.8-0.0}$	$< 2.0$
$\overline{B}^0 \rightarrow \pi^0 \overline{K}^0$	$5.8^{+5.5}_{-3.1}$	$9.6^{+5.5+8.4}_{-2.9-3.0}$	$11.5 \pm 1.0$
$\overline{B}^0 \rightarrow f_0 \overline{K}^0$	$8.1^{+3.1}_{-2.6}$	$8.1^{+3.1+0.0}_{-2.7-0.0}^a$	$11.3 \pm 3.6$

<sup>a</sup>Only the intermediate states  $K^- \rho^+$  and  $\pi^+ K^{*-}$  are taken into account; see Sec. III for details.

though the predicted branching ratios of  $(\phi, \omega, \rho^0, \eta', \pi^0) \overline{K}^0$  are consistent with the data within the theoretical and experimental uncertainties, there are sizable discrepancies between the SD theory and experiment for the central values of their branching ratios. This may call for the consideration of subleading power corrections such as the annihilation topology and/or FSI effects.

In Tables II and III we have included the SD theoretical uncertainties arising from the variation of the unitarity angle  $\gamma = (63 \pm 15)^\circ$ , the renormalization scale  $\mu$  from  $2m_b$  to  $m_b/2$ , quark masses (especially the strange quark mass which is taken to be  $m_s(2 \text{ GeV}) = 90 \pm 20 \text{ MeV}$ ) and form factors as mentioned before. To obtain the SD errors shown in Tables II and III, we first scan randomly the points in the allowed ranges of the above four parameters in two separated groups: the first one and the last three ones, and then add each error in quadrature. For example, for the decay  $\overline{B}^0 \rightarrow \eta' K_S$  we obtain  $2\mathcal{B} = (42.1^{+0.2+45.6}_{-0.2-19.4}) \times 10^{-6}$ ,  $\mathcal{A} = 1.77^{+0.30+0.22}_{-0.18-0.30}\%$  and  $S = 0.737^{+0.002+0.002}_{-0.038-0.004}$ , where the first error is due to the variation of  $\gamma$  and the second error comes from the uncertainties in the renormalization scale, the strange quark mass and the form factors.

From Table III we obtain the differences between the  $CP$  asymmetry  $S_f^{SD}$  induced at short distances and the measured  $S_{J/\psi K_S}$  to be

$$\begin{aligned}
\Delta S_{\phi K_S}^{SD} &= 0.02^{+0.00}_{-0.04}, & \Delta S_{\omega K_S}^{SD} &= 0.12^{+0.05}_{-0.06}, & \Delta S_{\rho^0 K_S}^{SD} &= -0.09^{+0.03}_{-0.07}, \\
\Delta S_{\pi^0 K_S}^{SD} &= 0.06^{+0.02}_{-0.04}, & \Delta S_{\eta' K_S}^{SD} &= 0.01^{+0.00}_{-0.04}, & \Delta S_{\eta K_S}^{SD} &= 0.07^{+0.02}_{-0.04}, \\
\Delta S_{f_0 K_S}^{SD} &= 0.02^{+0.00}_{-0.04}, & & & & 
\end{aligned} \tag{2.18}$$

where the experimental error of  $S_{J/\psi K_S}$  is not included. Our results for  $\Delta S_{\phi K_S}^{SD}$  and  $\Delta S_{\eta' K_S}^{SD}$  are consistent with that obtained in [19].<sup>2</sup> As expected before, the  $\omega K_S$  and  $\rho^0 K_S$  modes have the

[17] and hence  $\mathcal{B}(B^- \rightarrow f_0 K^-) \approx (15.9^{+3.8}_{-3.7}) \times 10^{-6}$ .

<sup>2</sup> Note that unlike [19] we did not include the theoretical uncertainties arising from power corrections. Otherwise, there will be a double counting problem when considering LD rescattering effects.

TABLE III: Short-distance (SD) and long-distance (LD) contributions to the time-dependent  $CP$  asymmetry. The first and second theoretical errors correspond to the SD and LD ones, respectively (see the text for details).

Final State	$-n_f S_f$			$\mathcal{A}_f(\%)$		
	SD	SD+LD	Expt	SD	SD+LD	Expt
$\phi K_S$	$0.747^{+0.002}_{-0.039}$	$0.759^{+0.007+0.005}_{-0.041-0.006}$	$0.35 \pm 0.20$	$1.4^{+0.3}_{-0.5}$	$-2.6^{+0.8+1.1}_{-1.0-0.9}$	$4 \pm 17$
$\omega K_S$	$0.850^{+0.052}_{-0.055}$	$0.736^{+0.022+0.025}_{-0.035-0.014}$	$0.55^{+0.30}_{-0.32}$	$-7.3^{+3.5}_{-2.6}$	$-13.2^{+3.9+2.1}_{-2.8-2.6}$	$48 \pm 25$
$\rho^0 K_S$	$0.635^{+0.028}_{-0.067}$	$0.761^{+0.071+0.073}_{-0.079-0.100}$	—	$9.0^{+2.2}_{-4.6}$	$46.6^{+12.9+10.8}_{-14.7-5.9}$	—
$\eta' K_S$	$0.737^{+0.002}_{-0.038}$	$0.725^{+0.004+0.005}_{-0.036-0.003}$	$0.43 \pm 0.11$	$1.8^{+0.4}_{-0.4}$	$2.1^{+0.6+0.5}_{-0.3-0.2}$	$4 \pm 8$
$\eta K_S$	$0.793^{+0.017}_{-0.044}$	$0.802^{+0.025+0.002}_{-0.046-0.004}$	—	$-6.1^{+5.1}_{-2.0}$	$-3.7^{+4.4+1.4}_{-1.8-2.4}$	—
$\pi^0 K_S$	$0.787^{+0.018}_{-0.044}$	$0.770^{+0.006+0.015}_{-0.042-0.019}$	$0.34^{+0.27}_{-0.29}$	$-3.4^{+2.1}_{-1.1}$	$3.7^{+1.8+2.0}_{-2.0-0.4}$	$-8 \pm 14$
$f_0 K_S$	$0.749^{+0.002}_{-0.039}$	$0.749^{+0.002+0.0}_{-0.039-0.0}^a$	$0.39 \pm 0.26$	$0.77^{+0.13}_{-0.10}$	$0.75^{+0.14+0.01}_{-0.09-0.01}^a$	$-14 \pm 22$

<sup>a</sup>Only the intermediate states  $K^-\rho^+$  and  $\pi^+K^{*-}$  are taken into account (see Sec. III for details). This means that the prediction of the LD effects on the  $f_0 K_S$  mode is less certain.

largest deviation of  $S_f$  from the naive expectation owing to the large tree pollution. In contrast, tree pollution in  $\eta' K_S$  is diluted by the prominent  $s\bar{s}$  content of the  $\eta'$ . As for direct  $CP$  violation, sizable direct  $CP$  asymmetries are predicted for  $\omega K_S$  and  $\rho^0 K_S$  based on SD contributions.

### III. LONG-DISTANCE CONTRIBUTIONS

As noticed in passing, the predicted branching ratios for the decays  $\overline{B}^0 \rightarrow (\phi, \omega, \rho^0, \pi^0) K_S$  by the short-distance QCD factorization approach are generally too low by a factor of 2 compared to experiment. Just like the pQCD approach [29] where the annihilation topology plays an essential role for producing sizable strong phases and for explaining the penguin-dominated  $VP$  modes, it has been suggested in [19] that a favorable scenario (denoted as S4) for accommodating the observed penguin-dominated  $B \rightarrow PV$  decays and the measured sign of direct  $CP$  asymmetry in  $\overline{B}^0 \rightarrow K^-\pi^+$  is to have a large annihilation contribution by choosing  $\rho_A = 1$ ,  $\phi_A = -55^\circ$  for  $PP$ ,  $\phi_A = -20^\circ$  for  $PV$  and  $\phi_A = -70^\circ$  for  $VP$  modes. The sign of  $\phi_A$  is chosen so that the direct  $CP$  violation  $A_{K^-\pi^+}$  agrees with the data. However, there are at least three difficulties with this scenario. First, the origin of these phases is unknown and their signs are not predicted. Second, since both annihilation and hard spectator scattering encounter endpoint divergences, there is no reason that soft gluon effects will only modify  $\rho_A$  but not  $\rho_H$ . Third, the annihilation topologies do not help enhance the  $\pi^0\pi^0$  and  $\rho^0\pi^0$  modes; both pQCD and QCDF approaches fail to describe these two color-suppressed tree-dominated modes. As stressed in [19], one would wish to have an explanation of the data without invoking weak annihilation.

As shown in [15], final-state rescattering can have significant effects on decay rates and  $CP$  violation. For example, the branching ratios of the penguin-dominated decay  $\phi K^{*-}$  can be enhanced from  $\sim 5 \times 10^{-6}$  predicted by QCDF to the level of  $1 \times 10^{-5}$  by FSIs via rescattering of charm

intermediate states [15]. Indeed, it has been long advocated that charming-penguin long-distance contributions increase significantly the  $B \rightarrow K\pi$  rates and yield better agreement with experiment [30, 31]. The color-suppressed modes  $D^0\pi^0$ ,  $\pi^0\pi^0$  and  $\rho^0\pi^0$  in  $B$  decays can also be easily enhanced by rescattering effects. Moreover, large nonperturbative strong phases can be generated from the final-state interactions through the absorptive part of rescattering amplitudes. We have shown explicitly in [15] that direct  $CP$ -violating partial rate asymmetries in  $K^-\pi^+$ ,  $\rho^+\pi^-$  and  $\pi^+\pi^-$  modes are significantly affected by final-state rescattering and their signs, which are different from what is expected from the short-distance QCDF approach, are correctly predicted. In order to avoid the double-counting problem, we will turn off the LD effects induced from the power corrections due to non-vanishing  $\rho_A$  and  $\rho_H$ ; that is, we set  $\rho_A = \rho_H = 0$  and  $\phi_A = \phi_H = 0$ , therefore it is important to note that we are not adding FSI on top of QCDF. We wish to stress that in principle LD rescattering effects can be included in the framework of QCDF, but that requires modelling of  $\Lambda_{\text{QCD}}/m_b$  power corrections and, in particular, one may then need to adopt non-vanishing values of  $\rho_A$ ,  $\rho_H$ ,  $\phi_A$  and  $\phi_H$  [19], as mentioned above. In this work, we are providing a specific model for final-state rescattering to complement QCDF.

Besides direct  $CP$  violation, the mixing-induced  $CP$  asymmetry  $S_f$  also could be affected by final-state rescattering from some intermediate states. When the intermediate states are charmless, the relevant CKM matrix element is  $V_{ub}V_{us}^* \approx A\lambda^4 R_b e^{-i\gamma}$  which carries the weak phase  $\gamma$ . In general, the charmless intermediate states will essentially not affect the decay rates but may have potentially sizable effect on  $S_f$ , whereas the charm intermediate states will affect both the branching ratios and  $S_f$ .

### A. Final-state rescattering

At the quark level, final-state rescattering can occur through quark exchange and quark annihilation. In practice, it is extremely difficult to reliably calculate the FSI effects, but it may become amenable to estimate these effects at the hadron level where FSIs manifest as the rescattering processes with  $s$ -channel resonances and one particle exchange in the  $t$ -channel. In contrast to  $D$  decays, the  $s$ -channel resonant FSIs in  $B$  decays is expected to be suppressed relative to the rescattering effect arising from quark exchange owing to the lack of the existence of resonances at energies close to the  $B$  meson mass. Therefore, we will model FSIs as rescattering processes of some intermediate two-body states with one particle exchange in the  $t$ -channel and compute the absorptive part of the rescattering amplitude via the optical theorem [15].

Given the weak Hamiltonian in the form  $H_W = \sum_i \lambda_i Q_i$ , where  $\lambda_i$  is the combination of the quark mixing matrix elements and  $Q_i$  is a  $T$ -even local operator ( $T$ : time reversal), the absorptive part of final-state rescattering can be obtained by using the optical theorem and time-reversal invariant weak decay operator  $Q_i$ . From the time reversal invariance of  $Q$  ( $= U_T Q^* U_T^\dagger$ ), it follows that

$$\langle i; \text{out} | Q | B; \text{in} \rangle^* = \sum_j S_{ji}^* \langle j; \text{out} | Q | B; \text{in} \rangle, \quad (3.1)$$

where  $S_{ij} \equiv \langle i; \text{out} | j; \text{in} \rangle$  is the strong interaction  $S$ -matrix element, and we have used

$U_T|\text{out}(\text{in})\rangle^* = |\text{in}(\text{out})\rangle$  to fix the phase convention. Eq. (3.1) implies an identity related to the optical theorem. Noting that  $S = 1 + iT$ , we find

$$2\mathcal{A}bs\langle i; \text{out}|Q|B; \text{in}\rangle = \sum_j T_{ji}^*\langle j; \text{out}|Q|B; \text{in}\rangle, \quad (3.2)$$

where use of the unitarity of the  $S$ -matrix has been made. Specifically, for two-body  $B$  decays, we have

$$\begin{aligned} \mathcal{A}bs M(p_B \rightarrow p_a p_b) &= \frac{1}{2} \sum_j \left( \Pi_{k=1}^j \int \frac{d^3 \vec{q}_k}{(2\pi)^3 2E_k} \right) (2\pi)^4 \\ &\times \delta^4(p_a + p_b - \sum_{k=1}^j q_k) M(p_B \rightarrow \{q_k\}) T^*(p_a p_b \rightarrow \{q_k\}). \end{aligned} \quad (3.3)$$

Thus the optical theorem relates the absorptive part of the two-body decay amplitude to the sum over all possible  $B$  decay final states  $\{q_k\}$ , followed by the strong rescattering  $\{q_k\} \rightarrow p_a p_b$ . In principle, the dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation

$$\mathcal{D}is A(m_B^2) = \frac{\mathcal{P}}{\pi} \int_s^\infty \frac{\mathcal{A}bs A(s')}{s' - m_B^2} ds'. \quad (3.4)$$

Unlike the absorptive part, it is known that the dispersive contribution suffers from the large uncertainties due to some possible subtractions and the complication from integrations. For this reason, we will assume the dominance of the rescattering amplitude by the absorptive part and ignore the dispersive part in the present work.

The relevant Lagrangian for final state strong interactions is given by

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_h, \quad (3.5)$$

where

$$\begin{aligned} \mathcal{L}_l &= -\frac{1}{4} \text{Tr}[F_{\mu\nu}(V)F^{\mu\nu}(V)] + ig_{VPP} \text{Tr}(V^\mu P \overleftrightarrow{\partial}_\mu P) + g_{VVP} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(\partial_\mu V_\nu \partial_\alpha V_\beta P) \\ &= -\frac{1}{4} \text{Tr}[f_{\mu\nu}(V)f^{\mu\nu}(V)] - i\frac{g_V}{2} (\phi_{\mu\nu} K^{*- \mu} K^{*+ \nu} + K_{\mu\nu}^{*-} K^{*+ \mu} \phi^\nu + K_{\mu\nu}^{*+} K^{*- \nu} \phi^\mu + \dots) \\ &\quad + ig_{VPP} \left[ \phi^\mu K^- \overleftrightarrow{\partial}_\mu K^+ + \rho^{+ \mu} (K^0 \overleftrightarrow{\partial}_\mu K^- - \sqrt{2} \pi^0 \overleftrightarrow{\partial}_\mu \pi^-) + K^{*- \mu} \pi^+ \overleftrightarrow{\partial}_\mu K^0 \right. \\ &\quad \left. + K^{*-} (\frac{1}{\sqrt{2}} \eta'_q \overleftrightarrow{\partial}_\mu K^+ + K^+ \overleftrightarrow{\partial}_\mu \eta'_s) + \overline{K}^{*0} (\frac{1}{\sqrt{2}} \eta'_q \overleftrightarrow{\partial}_\mu K^0 + K^0 \overleftrightarrow{\partial}_\mu \eta'_s) + \dots \right] \\ &\quad + g_{VVP} \epsilon^{\mu\nu\alpha\beta} \left[ K^- (\partial_\mu K_\nu^{*+} \partial_\alpha \phi_\beta) + K^+ (\partial_\mu \phi_\nu \partial_\alpha K_\beta^{*-}) + K^0 (\partial_\mu K_\nu^{*-} \partial_\alpha \rho_\beta^+) \right. \\ &\quad \left. + \sqrt{2} \pi^\pm (\partial_\mu \rho_\nu^\mp \partial_\alpha \omega_\beta) + \dots \right], \end{aligned} \quad (3.6)$$

with  $P$  and  $V$  being the usual pseudoscalar and vector multiplets, respectively,  $F_{\mu\nu} = f_{\mu\nu} + ig_V[V_\mu, V_\nu]/2$ ,  $f_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$  and

$$\begin{aligned} \mathcal{L}_h &= -ig_{D^*DP} (D^i \partial^\mu P_{ij} D_\mu^{*j\dagger} - D_\mu^{*i} \partial^\mu P_{ij} D^{j\dagger}) - \frac{1}{2} g_{D^*D^*P} \epsilon_{\mu\nu\alpha\beta} D_i^{*\mu} \partial^\nu P^{ij} \overleftrightarrow{\partial}^\alpha D_j^{*\beta\dagger} \\ &\quad - ig_{DDV} D_i^\dagger \overleftrightarrow{\partial}_\mu D^j (V^\mu)_j^i - 2f_{D^*DV} \epsilon_{\mu\nu\alpha\beta} (\partial^\mu V^\nu)_j^i (D_i^\dagger \overleftrightarrow{\partial}^\alpha D^{*\beta j} - D_i^{*\beta\dagger} \overleftrightarrow{\partial}^\alpha D^j) \\ &\quad + ig_{D^*D^*V} D_i^{*\nu\dagger} \overleftrightarrow{\partial}_\mu D_\nu^{*j} (V^\mu)_j^i + 4if_{D^*D^*V} D_{i\mu}^{*\dagger} (\partial^\mu V^\nu - \partial^\nu V^\mu)_j^i D_\nu^{*j}. \end{aligned} \quad (3.7)$$

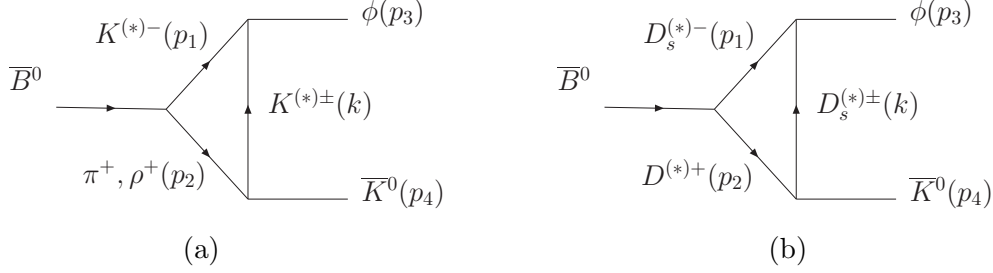


FIG. 1: Final-state rescattering contributions to the  $\bar{B}^0 \rightarrow \phi \bar{K}^0$  decay.

Only those terms relevant for later purposes are shown in  $\mathcal{L}_l$  and the convention  $\epsilon^{0123} = 1$  has been adopted. For the coupling constants, we take  $g_{\rho KK} \simeq g_{\rho\pi\pi} \simeq 4.28$ ,<sup>3</sup>  $g_{\phi KK} = 4.48$ ,  $\sqrt{2} g_{VVP} = 16 \text{ GeV}^{-1}$  [32]. In the chiral and heavy quark limits, we have [33]

$$g_{D^*D\pi} = \frac{g_{D^*D\pi}}{m_{D^*}}, \quad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \quad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \quad (3.8)$$

with  $f_\pi = 132 \text{ MeV}$ . The parameters  $g_V$ ,  $\beta$  and  $\lambda$  (not to be confused with the Wolfenstein parameter  $\lambda$ ) thus enter into the effective chiral Lagrangian describing the interactions of heavy mesons with low momentum light vector mesons (see e.g. [33]). The parameter  $g_V$  respects the relation  $g_V = m_\rho/f_\pi = 5.8$  [33]. We shall follow [34] to use  $\beta = 0.9$  and  $\lambda = 0.56 \text{ GeV}^{-1}$ . The coupling  $g_{D^*D\pi}$  has been extracted by CLEO to be  $17.9 \pm 0.3 \pm 1.9$  from the measured  $D^{*+}$  width [35].

### B. $B^0 \rightarrow \phi K_S$ as an example

We next proceed to study long-distance rescattering contributions to the  $b \rightarrow sq\bar{q}$  transition-induced decays  $\bar{B}^0 \rightarrow (\phi, \omega, \rho^0, \eta', \pi^0, f_0) K_S$ . To illustrate the calculations of rescattering amplitudes, we shall take the  $\phi K_S$  mode as an example. Its major final-state rescattering diagrams are depicted in Fig. 1.

The absorptive parts of the  $\bar{B}^0 \rightarrow K^- \rho^+ \rightarrow \phi \bar{K}^0$  amplitude via the  $K^\pm$ ,  $K^{*\pm}$  exchanges are given by

$$\begin{aligned} \text{Abs}(K^- \rho^+; K^\pm) &= \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^- \rho^+)}{2\varepsilon_2^* \cdot p_B} \\ &\quad \times \sum_{\lambda_2} 2\varepsilon_2^* \cdot p_B (-2i) g_{\phi KK} (\varepsilon_3^* \cdot p_1) \frac{F^2(t, m_K)}{t - m_K^2} 2i g_{\rho KK} (\varepsilon_2 \cdot p_4) \\ &= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^- \rho^+)}{2\varepsilon_2^* \cdot p_B} \\ &\quad \times 4g_{\phi KK} \frac{F^2(t, m_K)}{t - m_K^2} g_{\rho KK} (A_1^{(1)} - A_2^{(1)}) \left( -p_1 \cdot p_4 + \frac{(p_1 \cdot p_2)(p_2 \cdot p_4)}{m_2^2} \right), \quad (3.9) \end{aligned}$$

<sup>3</sup> The  $\rho\pi\pi$  coupling defined here differs from that in [15] by a factor of  $1/\sqrt{2}$ .

$$\begin{aligned}
Abs(K^-\rho^+; K^{*\pm}) &= \frac{1}{2} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^-\rho^+)}{2\varepsilon_2^* \cdot p_B} \\
&\times \sum_{\lambda_2} 2\varepsilon_2^* \cdot p_B (-i) g_{\phi K^* K} [p_1, \mu, p_3, \varepsilon_3^*] \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{K^*}^2} \right) \\
&\times \frac{F^2(t, m_{K^*})}{t - m_{K^*}^2} (-i) g_{\rho K^* K} [p_4, \nu, p_2, \varepsilon_2] \\
&= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^-\rho^+)}{2\varepsilon_2^* \cdot p_B} \\
&\times g_{\phi K^* K} \frac{F^2(t, m_{K^*})}{t - m_{K^*}^2} g_{\rho K^* K} (-2A_1^{(2)} m_3^2), \tag{3.10}
\end{aligned}$$

where the dependence of the polarization vector in the amplitude  $A(\bar{B}^0 \rightarrow K^-\rho^+)$  has been extracted and  $k \equiv p_1 - p_3$ . In order to avoid using too many dummy indices, we have defined  $[A, B, C, D] \equiv \epsilon_{\alpha\beta\gamma\delta} A^\alpha B^\beta C^\gamma D^\delta$ ,  $[A, B, C, \mu] \equiv \epsilon_{\alpha\beta\gamma\mu} A^\alpha B^\beta C^\gamma$  and so on for later convenience. Moreover, we have applied the identities [15]

$$\begin{aligned}
p_{1\mu} &\doteq P_\mu A_1^{(1)} + q_\mu A_2^{(1)}, \\
p_{1\mu} p_{1\nu} &\doteq g_{\mu\nu} A_1^{(2)} + P_\mu P_\nu A_2^{(2)} + (P_\mu q_\nu + q_\mu P_\nu) A_3^{(2)} + q_\mu q_\nu A_4^{(2)}, \tag{3.11}
\end{aligned}$$

under the the integration

$$\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) f(t) \times \{p_{1\mu}, p_{1\mu} p_{1\nu}\}, \tag{3.12}$$

with  $P \equiv p_3 + p_4$ ,  $q \equiv p_3 - p_4$ . These identities follow from the fact that the above integration can be expressed only in terms of the external momenta  $p_3, p_4$  with suitable Lorentz and permutation structures. The explicit expressions of  $A_j^{(i)} = A_j^{(i)}(t, m_B^2, m_1^2, m_2^2, m_3^2, m_4^2)$  can be found in [15].

Before proceeding it should be stressed that we have applied the hidden gauge symmetry Lagrangian Eq. (3.6) for light vector mesons and the chiral Lagrangian Eq. (3.7), based on heavy quark effective theory (HQET) and chiral symmetry, for heavy mesons to determine the strong vertices in Fig. 1. This requires that the involved light pseudoscalar or vector mesons be soft. However, the final state particles are necessarily hard and the particle exchanged in the  $t$  channel can be far off shell, especially for the  $t$ -exchanged  $D$  meson. This is beyond the applicability of the aforementioned chiral perturbation theory and HQET. Therefore, as stressed in [15], it is necessary to introduce the form factor  $F(t, m)$  appearing in Eqs. (3.9) and (3.10) to take care of the off-shell effect of the  $t$ -channel exchanged particle and the hardness of the final particles. Indeed, if the off-shell effect is not considered, the long-distance rescattering contributions will become so large that perturbation theory is no longer trustworthy. For example, since  $B \rightarrow D_s \bar{D}$  is CKM doubly enhanced relative to  $B \rightarrow K\pi$ , the rescattering process  $B \rightarrow D_s \bar{D} \rightarrow K\pi$  will overwhelm the initial  $B \rightarrow K\pi$  amplitude. Hence, form factors or cutoffs must be introduced to the strong vertices to render the calculation meaningful in perturbation theory.

The form factor  $F(t, m)$  is usually parametrized as

$$F(t, m) = \left( \frac{\Lambda^2 - m^2}{\Lambda^2 - t} \right)^n, \tag{3.13}$$

normalized to unity at  $t = m^2$  with  $\Lambda$  being a cutoff parameter which should be not far from the physical mass of the exchanged particle. To be specific, we write

$$\Lambda = m_{\text{exc}} + \eta \Lambda_{\text{QCD}}, \quad (3.14)$$

where the parameter  $\eta$  is expected to be of order unity and it depends not only on the exchanged particle but also on the external particles involved in the strong-interaction vertex. The monopole behavior of the form factor (i.e.  $n = 1$ ) is preferred as it is consistent with the QCD sum rule expectation [37]. Although the strong couplings are large in the magnitude, the rescattering amplitude is suppressed by a factor of  $F^2(t) \sim m^2 \Lambda_{\text{QCD}}^2 / t^2$ . Consequently, the off-shell effect will render the perturbative calculation meaningful. Moreover, since in the heavy quark limit  $t \sim m_B^2$ , the final-state rescattering amplitude does vanish in the  $m_B \rightarrow \infty$  limit, as it should.

Likewise, the absorptive part of the  $\bar{B}^0 \rightarrow K^{*-} \pi^+ \rightarrow \phi \bar{K}^0$  amplitude via the  $K^{*\pm}$  exchange is given by

$$\begin{aligned} \mathcal{A}bs(K^{*-} \pi^+; K^{*\pm}) &= \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^{*-} \pi^+)}{2\varepsilon_1^* \cdot p_B} \\ &\quad \times \sum_{\lambda_1} 2\varepsilon_1^* \cdot p_B i \frac{g_V}{2} [\varepsilon_{3\mu}^* (2\varepsilon_1 \cdot p_3) - \varepsilon_1 \cdot \varepsilon_3^* (p_1 + p_3)_\mu + \varepsilon_{1\mu} (2p_1 \cdot \varepsilon_3)] \\ &\quad \times \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{K^*}^2} \right) \frac{F^2(t, m_{K^*})}{t - m_{K^*}^2} (-i) g_{K^* K \pi} (p_2 + p_4)_\nu \\ &= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^{*-} \pi^+)}{2\varepsilon_1^* \cdot p_B} \\ &\quad \times g_V \frac{F^2(t, m_{K^*})}{2(t - m_{K^*}^2)} g_{K^* K \pi} \left\{ 2 \left[ 2 - A_1^{(1)} + A_2^{(1)} + (A_1^{(1)} - A_2^{(1)}) \frac{m_2^2 - m_4^2}{m_{K^*}^2} \right] \right. \\ &\quad \times \left( p_2 \cdot p_3 - \frac{(p_1 \cdot p_2)(p_2 \cdot p_3)}{m_1^2} \right) + \left[ A_1^{(1)} - A_2^{(1)} - 1 + (A_1^{(1)} - A_2^{(1)}) \frac{p_1 \cdot p_2}{m_1^2} \right] \\ &\quad \times \left[ (p_1 + p_3) \cdot (p_2 + p_4) + \frac{1}{m_{K^*}^2} (m_1^2 - m_3^2)(m_2^2 - m_4^2) \right] + 2(A_1^{(1)} - A_2^{(1)}) \\ &\quad \times \left[ \left( p_2 \frac{m_{K^*}^2 - m_2^2 + m_4^2}{m_{K^*}^2} + p_4 \frac{m_{K^*}^2 + m_2^2 - m_4^2}{m_{K^*}^2} \right) \cdot \left( p_2 - p_1 \frac{p_1 \cdot p_2}{m_1^2} \right) \right] \left. \right\}. \end{aligned} \quad (3.15)$$

Note that since the  $\pi K K$  vertex is absent, there is no contribution from the  $K^+$  exchanged particles.

The  $\bar{B}^0 \rightarrow \phi \bar{K}^0$  decay also receives contributions from charmless  $VV$  modes. The leading candidate is the  $K^{*-} \rho^+$  mode via the  $p$ -wave configuration. However, as we have checked numerically, its amplitude is one or two orders of magnitude smaller than those from the previous two charmless  $VP$  modes and its effect on  $S_f$  is quite small. Given this, we will not go into any further detail on the rescattering from charmless  $VV$  modes. Thus far we have only considered the contributions where the two intermediate states originating from the weak vertex are on shell. There are additional contributions where one of the mesons coming from the weak vertex and the exchanged meson in the  $t$ -channel are on shell. For example, in the diagram of Fig. 1(a), we can set  $K^-$  and  $K^+$  on shell while keeping  $\rho^+$  off shell. This corresponds to the 3-body weak decay  $\bar{B}^0 \rightarrow K^+ K^- \bar{K}^0$  followed by the strong rescattering  $K^+ K^- \bar{K}^0 \rightarrow \phi \bar{K}^0$  where  $\bar{K}^0$  behaves as a spectator. However, there are



many possible pole contributions to  $\bar{B}^0 \rightarrow K^+ K^- \bar{K}^0$ . In addition to  $\bar{B}^0 \rightarrow K^- \rho^+ \rightarrow K^+ K^- \bar{K}^0$  as inferred from Fig. 1(a), one can also have  $\bar{B}^0 \rightarrow \bar{B}_s^* \bar{K}^0 \rightarrow K^+ K^- \bar{K}^0$ , for example. In the present work, we will only focus on two-body intermediate state contributions to the absorptive part. Since the analogous 3-body contributions do not occur in Fig. 1(b) and since Fig. 1(a) is CKM doubly suppressed relative to Fig. 1(b), it is safe to neglect the additional three-body contributions for our purposes.

We next turn to the FSI contribution arising from the intermediate states  $D_s^{(*)-} D^{(*)}$ . Note that only the  $p$ -wave configurations of the  $D_s^{(*)-} D^{(*)}$  systems can rescatter into the  $\phi \bar{K}^0$  final state. The absorptive parts of  $\bar{B}^0 \rightarrow D_s^{*-} D^+ \rightarrow \phi \bar{K}^0$  amplitudes via  $D_s^*$  exchanges,  $\bar{B} \rightarrow D_s^{*-} D^+$ ,  $D_s^{*-} D^{*+} \rightarrow \phi \bar{K}^0$  amplitudes via  $D_s$  and  $D_s^*$  exchanges, are given by

$$\begin{aligned}
Abs(D_s^{*-} D^+; D_s^{\pm}) &= \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow D_s^{*-} D^+)}{2\varepsilon_1^* \cdot P} \\
&\quad \times \sum_{\lambda_1} 2\varepsilon_1^* \cdot P (-4i) f_{D_s^* D_s^* \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} i g_{D_s^* DK} \\
&\quad \times \varepsilon_1^\rho (\sigma g_{\rho\mu} p_1 \cdot \varepsilon_3^* + p_{3\rho} \varepsilon_{3\mu}^* - \varepsilon_{3\rho}^* p_{3\mu}) \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{D_s^*}^2} \right) p_{4\nu} \\
&= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \\
&\quad \times \frac{A(\bar{B}^0 \rightarrow D_s^{*-} D^+)}{2\varepsilon_1^* \cdot P} 4 f_{D_s^* D_s^* \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} g_{D_s^* DK} \\
&\quad \times \left\{ \sigma(A_1^{(1)} - A_2^{(1)}) \left( p_2 - \frac{p_1 \cdot p_2}{m_1^2} p_1 \right) \cdot \left( p_4 - k \frac{k \cdot p_4}{m_{D_s^*}^2} \right) \right. \\
&\quad \left. + \left[ 1 - (A_1^{(1)} - A_2^{(1)}) \frac{k \cdot p_4}{m_{D_s^*}^2} \right] \left( p_2 \cdot p_3 - \frac{p_1 \cdot p_2}{m_1^2} \frac{p_1 \cdot p_3}{m_1^2} \right) \right. \\
&\quad \left. - \left[ 1 - (A_1^{(1)} - A_2^{(1)}) \frac{P \cdot p_1}{m_1^2} \right] \left( p_3 \cdot p_4 - \frac{p_3 \cdot k}{m_{D_s^*}^2} \frac{k \cdot p_4}{m_{D_s^*}^2} \right) \right\}, \\
Abs(D_s^- D^{*+}; D_s^\pm) &= \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow D_s^- D^{*+})}{2\varepsilon_2^* \cdot P} \\
&\quad \times \sum_{\lambda_2} 2\varepsilon_2^* \cdot P 2i g_{D_s D_s \phi} \varepsilon_3^* \cdot p_1 \frac{F^2(t, m_{D_s})}{t - m_{D_s}^2} (-i) g_{D^* D_s K} p_4 \cdot \varepsilon_2 \\
&= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow D_s^- D^{*+})}{2\varepsilon_2^* \cdot P} \\
&\quad \times \left\{ 2g_{D_s D_s \phi} \frac{F^2(t, m_{D_s})}{t - m_{D_s}^2} g_{D^* D_s K} (A_1^{(1)} - A_2^{(1)}) \left( -p_1 \cdot p_4 + \frac{p_1 \cdot p_2}{m_2^2} \frac{p_2 \cdot p_4}{m_2^2} \right) \right\}, \\
Abs(D_s^- D^{*+}; D_s^{\pm}) &= \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow D_s^- D^{*+})}{2\varepsilon_2^* \cdot P} \\
&\quad \times \sum_{\lambda_2} 2\varepsilon_2^* \cdot P 4i f_{D_s^* D_s \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} i g_{D^* D_s^* K}
\end{aligned}$$

$$\begin{aligned}
& \times [p_3, \varepsilon_3^*, p_1, \mu] \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{D_s^*}^2} \right) [\varepsilon_2, p_4, p_2, \nu] \\
& = 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\overline{B}^0 \rightarrow D_s^- D^{*+})}{2\varepsilon_2^* \cdot P} \\
& \quad \times (-8) f_{D_s^* D_s \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} g_{D^* D_s^* K} m_3^2 A_1^{(2)}, \\
Abs(D_s^{*-} D^{*+}; D_s^\pm) & = \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (ic[\mu, \nu, P, p_2]) \\
& \quad \times \sum_{\lambda_1, \lambda_2} \varepsilon_1^{*\mu} \varepsilon_2^{*\nu} (-4i) f_{D_s^* D_s \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} (-i) g_{D^* D_s K} [p_3, \varepsilon_3^*, p_1, \varepsilon_1] p_4 \cdot \varepsilon_2 \\
& = 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \\
& \quad \times (-4ic) f_{D_s^* D_s \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} g_{D^* D_s K} m_3^2 A_1^{(2)}, \\
Abs(D_s^{*-} D^{*+}; D_s^{\pm\pm}) & = \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (ic[\delta, \beta, P, p_2]) \\
& \quad \times \sum_{\lambda_1, \lambda_2} \varepsilon_1^{*\delta} \varepsilon_2^{*\beta} (-4i) f_{D_s^* D_s \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} i g_{D^* D_s^* K} \\
& \quad \times \varepsilon_1^\rho (\sigma g_{\rho\mu} p_1 \cdot \varepsilon_3^* + p_{3\rho} \varepsilon_{3\mu}^* - \varepsilon_{3\rho}^* p_{3\mu}) \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{D_s^*}^2} \right) [\varepsilon_2, p_4, p_2, \nu] \\
& = 2\varepsilon_3^* \cdot p_B \times \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \\
& \quad \times (-4ic) f_{D_s^* D_s \phi} \frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} g_{D^* D_s^* K^*} \\
& \quad \times [\sigma(A_1^{(1)} - A_2^{(1)})(p_1 \cdot p_4 m_2^2 - p_1 \cdot p_2 p_2 \cdot p_4) + m_3^2 A_1^{(2)}], \tag{3.16}
\end{aligned}$$

where the dependence of the polarization vectors in  $A(\overline{B}^0 \rightarrow D_s^{*-} D^+, D_s^- D^{*+})$  has been extracted out explicitly and  $\sigma \equiv g_{D^* D^* V} / (2f_{D^* D^* V})$  and the  $\overline{B} \rightarrow D_s^{*-} D^{*+}$  decay amplitude has been denoted as

$$A(\overline{B} \rightarrow D_s^*(p_1, \lambda_1) D^*(p_2, \lambda_2)) = \varepsilon_1^{*\mu} \varepsilon_2^{*\nu} (a g_{\mu\nu} + b P_\mu P_\nu + ic[\mu, \nu, P, p_2]). \tag{3.17}$$

In order to perform a numerical study of the above analytic results, we need to specify the short-distance  $A(\overline{B} \rightarrow D_s^{(*)} D^{(*)})$  amplitudes. In the factorization approach, we have

$$\begin{aligned}
A(\overline{B} \rightarrow D_s^* D)_{SD} & = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 f_{D_s^*} m_{D_s^*} F_1^{BD}(m_{D_s^*}^2) (2\varepsilon_{D_s^*}^* \cdot p_B), \\
A(\overline{B} \rightarrow D_s D^*)_{SD} & = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 f_{D_s} m_{D^*} A_0^{BD^*}(m_{D_s}^2) (2\varepsilon_{D^*}^* \cdot p_B), \\
A(\overline{B} \rightarrow D_s^* D^*)_{SD} & = -i \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 f_{D_s^*} m_{D_s^*} (m_B + m_{D^*}) \varepsilon_{D_s^*}^{*\mu} \varepsilon_{D^*}^{*\nu} \left[ A_1^{BD^*}(m_{D_s^*}^2) g_{\mu\nu} \right. \\
& \quad \left. - \frac{2A_2^{BD^*}(m_{D_s^*}^2)}{(m_B + m_{D^*})^2} p_{B\mu} p_{B\nu} - i \frac{2V^{BD^*}(m_{D_s^*}^2)}{(m_B + m_{D^*})^2} \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D^*}^\beta \right]. \tag{3.18}
\end{aligned}$$

Since the phase of the parameter  $a_1$  originating from vertex corrections [see Eq. (2.5)] is very small, one can neglect the strong phase of the short-distance amplitudes.

The long-distance contribution to the  $\bar{B}^0 \rightarrow \omega \bar{K}^0$  decay can be preformed similarly. Due to the absence of the  $PPP$  vertex and the  $G$ -parity argument, the number of FSI diagrams from charmless intermediate states is greatly reduced compared to the previous case. For example, the  $K^-\rho^+$  intermediate state does not contribute to the  $\bar{K}^0\omega$  amplitude (through  $t$ -channel  $\pi$  and  $\rho$  exchanges) as both  $KK\pi$  and  $\rho\rho\omega$  vertices are forbidden. In fact, there is only one relevant rescattering diagram, namely,  $\bar{B}^0 \rightarrow K^{*-}\pi^+ \rightarrow \bar{K}^0\omega$  via  $\rho^\pm$  exchange, arising from charmless intermediate states and the corresponding absorptive part is given by

$$\begin{aligned}
\mathcal{A}bs(K^{*-}\pi^+; \rho^\pm) &= \frac{1}{2} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^{*-}\pi^+)}{2\varepsilon_1^* \cdot p_B} \\
&\quad \times \sum_{\lambda_1} 2\varepsilon_1^* \cdot p_B (ig_{\rho K^* K})(i\sqrt{2}g_{\omega\rho\pi}) [p_1, \varepsilon_1, k, \mu][k, \nu, p_4, \varepsilon_4^*] \\
&\quad \times \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{K^*}^2} \right) \frac{F^2(t, m_\rho)}{t - m_\rho^2} \\
&= 2\varepsilon_4^* \cdot p_B \times \frac{1}{2} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow K^{*-}\pi^+)}{2\varepsilon_1^* \cdot p_B} \\
&\quad \times (-2\sqrt{2}) g_{\rho K^* K} \frac{F^2(t, m_\rho)}{(t - m_\rho^2)} g_{\omega\rho\pi} m_4^2 A_1^{(2)}. \tag{3.19}
\end{aligned}$$

Furthermore, we have checked numerically that the  $\bar{B}^0 \rightarrow K^{*-}\rho^+ \rightarrow \bar{K}^0\omega$  contribution is small. Hence we will skip further detail on the rescattering from charmless  $VV$  modes.

The rescattering amplitudes of  $\bar{B}^0 \rightarrow D_s^{*-}D^+$ ,  $D^-D^{*+}$ ,  $D^{*-}D^{*+} \rightarrow \bar{K}^0\omega$  amplitudes via  $D$ ,  $D^*$  exchanges can be evaluated in a similar way. The analytic expressions of their absorptive parts are similar to those for  $\phi\bar{K}^0$ . For example,

$$\begin{aligned}
\mathcal{A}bs(D_s^{*-}D^+; D^\pm) &= 2\varepsilon_4^* \cdot p_B \times \frac{1}{2} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \frac{A(\bar{B}^0 \rightarrow D_s^{*-}D^{*+})}{2\varepsilon_2^* \cdot P} \\
&\quad \times \left\{ \sqrt{2}g_{D_s^*DK} \frac{F^2(t, m_D)}{t - m_D^2} g_{DD\omega} (1 - A_1^{(1)} - A_2^{(1)}) \left( -p_2 \cdot p_3 + \frac{p_1 \cdot p_2 p_1 \cdot p_3}{m_1^2} \right) \right\}, \tag{3.20}
\end{aligned}$$

is the same as  $\mathcal{A}bs(D_s^-D^{*+}; D_s^\pm)$  in (3.16) after the replacements  $g_{D^*D_sK} \rightarrow g_{D_s^*DK}$ ,  $f_{D_sD_s\phi} \rightarrow f_{DD\omega}/\sqrt{2}$ ,  $p_1 \leftrightarrow p_2$ ,  $p_3 \leftrightarrow p_4$ ,  $A_1^{(1)} - A_2^{(1)} \rightarrow 1 - A_1^{(1)} - A_2^{(1)}$  and a suitable replacement of the source amplitude (note that the replacement of momentum should not be performed in  $A_j^{(i)}$ ). Likewise, we can obtain  $\mathcal{A}bs(D_s^{*-}D^+; D^{*\pm})$ ,  $\mathcal{A}bs(D_s^-D^{*+}; D^{*\pm})$ ,  $\mathcal{A}bs(D_s^{*-}D^{*+}, D^{(*)\pm})$  from  $\mathcal{A}bs(D_s^-D^{*+}; D_s^{*\pm})$ ,  $\mathcal{A}bs(D_s^{*-}D^+; D_s^{*\pm})$ ,  $\mathcal{A}bs(D_s^{*-}D^{*+}, D_s^{(*)\pm})$  in (3.16), respectively, with  $A_1^{(2)}$  being unchanged and an additional overall minus sign for  $D_s^{*-}D^{*+}$  contributions. This similarity is by no means accidental; it follows from the so-called CPS symmetry, i.e. CP plus  $s \leftrightarrow d$  switch symmetry. A similar but more detailed discussion is given in [15] for the case of  $B \rightarrow \phi K^*$  decay.

The final-state rescattering contributions to other penguin-dominated decays  $B^0 \rightarrow (\rho^0, \eta^{(\prime)}, f_0)K_S$  can be worked out in a similar manner. The dominant intermediates states for each decay modes are summarized in Table IV.

TABLE IV: Dominant intermediate states contributing to various final states. For the  $\eta K_S$  final state, the intermediate states are the same as that for  $\eta' K_S$ .

Final state	Intermediate state (exchanged particle)
$\phi K_S$	$K^{*-}\pi^+(K^*), K^-\rho^+(K, K^*),$ $D_s^{*-}D^+(D_s^*), D_s^-D^{*+}(D_s, D_s^*), D_s^{*-}D^{*+}(D_s, D_s^*)$
$\omega K_S$	$K^{*-}\pi^+(\rho),$ $D_s^{*-}D^+(D, D^*), D_s^-D^{*+}(D^*), D_s^{*-}D^{*+}(D, D^*)$
$\rho^0 K_S$	$K^{*-}\pi^+(\rho),$ $D_s^{*-}D^+(D, D^*), D_s^-D^{*+}(D^*), D_s^{*-}D^{*+}(D, D^*)$
$\eta' K_S$	$\eta'\bar{K}^0(K^{*0}), K^{*-}\rho^+(K, K^*)$ $D_s^-D^+(D^*, D_s^*), D_s^{*-}D^{*+}(D, D_s, D^*, D_s^*)$
$\pi^0 K_S$	$\eta'\bar{K}^0(K^{*0})$ $D_s^-D^+(D^*), D_s^{*-}D^{*+}(D, D^*)$
$f_0 K_S$	$\rho^+K^-(K, \rho), \pi^+K^{*-}(\pi)$ $D_s^{*-}D^+(D, D_s^*), D_s^-D^{*+}(D^*)$

### C. Results and discussions

Writing  $A = A^{SD} + i\mathcal{A}bsA^{LD}$  with  $\mathcal{A}bsA^{LD}$  obtained above and the form factors given in [24], the results of the final-state rescattering effects on decay rates, direct and mixing-induced  $CP$  violation parameters are shown in Tables II and III. As pointed out in [15], the long-distance rescattering effects are sensitive to the cutoff parameter  $\Lambda$  appearing in Eq. (3.14) or  $\eta$  in Eq. (3.14). Since we do not have first-principles calculations of  $\eta$ , we will determine it from the measured branching ratios and then use it to predict the  $CP$ -violating parameters  $\mathcal{A}_f$  and  $S_f$ . As shown in [15],  $\eta = 0.69$  for the exchanged particles  $D$  and  $D^*$  is obtained from fitting to the  $B \rightarrow K\pi$  rates. We take  $\eta = 0.85$  for the exchanged particles  $D_{(s)}$  and  $D_{(s)}^*$  to fit the data of  $\bar{B}^0 \rightarrow \eta'\bar{K}^0$  rates and  $\eta = 1$  for other light exchanged particles such as  $\pi, K, K^*$ . As for the  $VP$  modes, namely,  $\phi K_S, \omega K_S$  and  $\rho^0 K_S$ , we take  $\eta_{D_{(s)}} = \eta_{D_{(s)}^*} = 0.95, 1.4$  and  $1.5$  respectively in order to accommodate their rates<sup>4</sup>. Note that the result  $\eta_{D_{(s)}} = \eta_{D_{(s)}^*} = 1.5$  for  $\rho^0 K_S$  is consistent with the value of  $\eta_D = \eta_{D^*} = 1.6$  obtained for  $B \rightarrow \rho\pi$  decays [15] within SU(3) symmetry. For the decay  $\bar{B}^0 \rightarrow f_0 K_S$ , since only the strong couplings of  $f_0$  to  $K\bar{K}$  and  $\pi\pi$  are available experimentally, we shall only consider the intermediate states  $K^-\rho^+$  and  $\pi^+K^{*-}$ . The LD theoretical uncertainties shown in Tables II and III originate

<sup>4</sup> Note that a monopole momentum dependence is used for the form factors throughout this paper [see Eq. (3.13)]. If a dipole form of the momentum dependence is used for  $D^*$ -exchange diagrams as in [15], we will obtain  $\mathcal{B}^{SD+LD}(\phi K^0) = (6.1_{-1.9-0.2}^{+2.0+0.4}) \times 10^{-6}$ ,  $S_{\phi K_S}^{SD+LD} = 0.723_{-0.036-0.012}^{+0.004+0.009}$ ,  $\mathcal{A}_{\phi K_S}^{SD+LD} = -0.070_{-0.017}^{+0.015} \pm 0.019$ ,  $\mathcal{B}^{SD+LD}(\omega K^0) = (2.5_{-1.4-0.3}^{+3.7+0.6}) \times 10^{-6}$ ,  $S_{\omega K_S}^{SD+LD} = 0.847_{-0.054-0.013}^{+0.029+0.007}$  and  $\mathcal{A}_{\omega K_S}^{SD+LD} = 0.013_{-0.078-0.029}^{+0.034+0.027}$ .

from three sources: an assignment of 15% error in  $\Lambda_{\text{QCD}}$ , the measured error in the coupling  $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$  [35] and 5% error in the form factors for  $B$  to  $D^{(*)}$  transitions. They are obtained by scanning randomly the points in the allowed ranges of the above-mentioned three parameters. The calculations of hadronic diagrams for FSI also involve many other theoretical uncertainties some of which are already discussed in [15]. From Tables II and III it is clear that the SD errors are in general not significantly affected by FSI effects and that LD uncertainties are in general comparable to the SD ones.

We see from Table II that final-state rescattering will enhance the decay rates of  $\phi K_S, \omega K_S, \rho^0 K_S, \eta' K_S$  and  $\pi^0 K_S$  but it does not affect the  $\eta K_S$  rate. The seemingly large disparity between  $\eta' K_S$  and  $\eta K_S$  for FSI can be understood as follows. There are two types of exchanged particles in the rescattering processes, namely,  $D^{(*)}$  and  $D_s^{(*)}$  (see Table IV). The former (latter) couple to the  $d$  ( $s$ ) quark component of the  $\eta^{(\prime)}$ . Since the  $\eta'$  and  $\eta$  wave functions are approximately given by Eq. (2.16), it is clear that the rescattering amplitudes due to the exchanged particles  $D^{(*)}$  and  $D_s^{(*)}$  interfere constructively for the  $\eta' K_S$  production but compensate largely for  $\eta K_S$ .

It should be stressed that although we have used the measured branching ratio of  $\eta' K^0$  to fix the LD contributions and the unknown cutoff parameter  $\eta$ , there exist some other possible mechanisms that can help explain its large rate. For example, the QCD anomaly effect manifested in the two gluon coupling with the  $\eta'$  may provide a dynamical enhancement of the  $\eta' K^0$  production [36]. And it is likely that both final-state rescattering and the gluon anomaly are needed to account for the unexpectedly large branching fraction of  $\eta' K$ . Note that both contributions carry negligible  $CP$ -odd phase. Hence, whether the anomalously large branching ratio of  $\eta' K$  comes from the QCD anomaly and/or from final-state rescattering, it will be very effective in diluting the  $u\bar{u}$  tree contributions and rendering  $\Delta S_{\eta' K_S}$  small.

We are not able to estimate the long-distance rescattering contributions to the  $f_0 K_S$  rate from intermediate charm states due to the absence of information on  $f_0 DD$  and  $f_0 D_{(s)}^* D_{(s)}^*$  couplings.

Since the modes  $\omega K_S, \rho^0 K_S, \phi K_S$  and  $\pi^0 K_S$  receive significant final-state rescattering contributions, it is natural to expect that their direct  $CP$  asymmetries will be affected accordingly. It is clear from Table III that the signs of  $\mathcal{A}_f$  in the last two channels are flipped by final-state interactions. As for the mixing-induced  $CP$  violation  $S_f$ , we see from Table III that  $\omega K_S$  and  $\rho^0 K_S$  receive the largest corrections from final-state rescattering, while the long-distance correction to  $\phi K_S$  is not as large as what was originally expected. The underlying reason is as follows. The mixing  $CP$  asymmetries  $S_{\omega K_S}^{SD}$  and  $S_{\rho^0 K_S}^{SD}$  deviate from  $\sin 2\beta$  as they receive contributions from the tree amplitude. The contribution from the tree amplitude is relatively enhanced as the QCD penguin amplitude is suppressed by a cancellation between penguin terms ( $|a_4 - r_\chi a_6| \ll |a_4|, |a_6|$ ). Final-state rescattering from  $D_s^{(*)} D^{(*)}$  states with vanishing weak phases will dilute the SD tree contribution and bring the asymmetry closer to  $\sin 2\beta$ . For the  $\phi K_S$  mode, the asymmetry  $S_{\phi K_S}^{SD}$  is slightly greater than  $\sin 2\beta$ . With rescattering from either charmless intermediate states, such as  $K^* \pi$ , or from  $D_s^{(*)} D^{(*)}$  states, the asymmetry is reduced to  $S_{\phi K_S} \simeq 0.73$ . When both charmless and charmful intermediate states are considered, the asymmetry is enhanced to  $S_{\phi K_S} \simeq 0.76$  owing to the interference effect from these two contributions.

Among the seven modes  $(\phi, \omega, \rho^0, \pi^0, \eta^{(\prime)}, f_0) K_S$ , the first three are the states where final-state

TABLE V: Direct  $CP$  asymmetry parameter  $\mathcal{A}_f$  and the mixing-induced  $CP$  parameter  $\Delta S_f^{SD+LD}$  for various modes. The first and second theoretical errors correspond to the SD and LD ones, respectively (see the text for details). The  $f_0 K_S$  channel is not included as we cannot make reliable estimate of FSI effects on this decay.

Final State	$\Delta S_f$			$\mathcal{A}_f(\%)$		
	SD	SD+LD	Expt	SD	SD+LD	Expt
$\phi K_S$	$0.02^{+0.00}_{-0.04}$	$0.03^{+0.01+0.01}_{-0.04-0.01}$	$-0.38 \pm 0.20$	$1.4^{+0.3}_{-0.5}$	$-2.6^{+0.8+0.0}_{-1.0-0.4}$	$4 \pm 17$
$\omega K_S$	$0.12^{+0.05}_{-0.06}$	$0.01^{+0.02+0.02}_{-0.04-0.01}$	$-0.17^{+0.30}_{-0.32}$	$-7.3^{+3.5}_{-2.6}$	$-13.2^{+3.9+1.4}_{-2.8-1.4}$	$48 \pm 25$
$\rho^0 K_S$	$-0.09^{+0.03}_{-0.07}$	$0.04^{+0.09+0.08}_{-0.10-0.11}$	—	$9.0^{+2.2}_{-4.6}$	$46.6^{+12.9+3.9}_{-13.7-2.6}$	—
$\eta' K_S$	$0.01^{+0.00}_{-0.04}$	$0.00^{+0.00+0.00}_{-0.04-0.00}$	$-0.30 \pm 0.11$	$1.8^{+0.4}_{-0.4}$	$2.1^{+0.5+0.1}_{-0.2-0.1}$	$4 \pm 8$
$\eta K_S$	$0.07^{+0.02}_{-0.04}$	$0.07^{+0.02+0.00}_{-0.05-0.00}$	—	$-6.1^{+5.1}_{-2.0}$	$-3.7^{+4.4+1.4}_{-1.8-2.4}$	—
$\pi^0 K_S$	$0.06^{+0.02}_{-0.04}$	$0.04^{+0.02+0.01}_{-0.03-0.01}$	$-0.39^{+0.27}_{-0.29}$	$-3.4^{+2.1}_{-1.1}$	$3.7^{+3.1+1.0}_{-1.7-0.4}$	$-8 \pm 14$

interactions may have a potentially large effect on the mixing-induced  $CP$  asymmetry  $S_f$ . In order to maximize the effects of FSIs on  $S_f$ , one should consider rescattering from charmless intermediate states that receive sizable tree contributions. Most of the intermediate states such as  $\bar{K}^0 \eta', K^{*-} \rho^+, \dots$ , in  $B$  decays are penguin dominated and hence will not affect  $S_f$ . For the decay  $\bar{B}^0 \rightarrow \phi K_S$ , we have rescattering from  $K^- \rho^+$  and  $K^{*-} \pi^+$ . Because of the absence of the penguin chiral enhancement in  $\bar{B}^0 \rightarrow K^{*-} \pi^+$  and the large cancellation between  $a_4$  and  $a_6$  penguin terms in  $\bar{B}^0 \rightarrow K^- \rho^+$ , it follows that the color-allowed tree contributions in these two modes are either comparable to or slightly smaller than the penguin effects. As for the  $\omega K_S$  mode, there is only one rescattering diagram, namely,  $\bar{B}^0 \rightarrow K^{*-} \pi^+ \rightarrow \bar{K}^0 \omega$ , arising from the charmless intermediate states. (The rescattering diagram from  $K^{*-} \rho^+$  is suppressed as elucidated before for the  $\phi K_S$  mode.) As a result, one will expect that final-state rescattering effect on  $S_f$  will be most prominent in  $B \rightarrow \omega K_S$ ,  $\rho^0 K_S$  and  $\phi K_S$ . Indeed, we see from Table III that FSI lowers  $S_{\omega K_S}$  by 15% and enhances  $S_{\rho^0 K_S}$  by 17% and  $S_{\phi K_S}$  slightly. The theoretical predictions and experimental measurements for the differences between  $S_f^{SD+LD}$  and  $S_{J/\psi K_S}$ ,  $\Delta S_f^{SD+LD}$ , are summarized in Table V. It is evident that final-state interactions cannot induce large  $\Delta S_f$  in any of these modes.

It is interesting to study the correlation between  $\mathcal{A}_f$  and  $S_f$  for the penguin dominated modes in the presence of FSIs. It follows from Eq. (1.8) that

$$\frac{\Delta S_f}{\mathcal{A}_f} = \cos 2\beta \cot \delta_f \approx 0.95 \cot \delta_f, \quad (3.21)$$

for  $r_f = (\lambda_u A_f^u)/(\lambda_c A_f^c) \ll 1$ . This ratio is independent of  $|r_f|$  and hence it is less sensitive to hadronic uncertainties. Therefore, it may provide a better test of the SM even in the presence of FSIs. Writing  $A_f^{u,c} = |A_{SD}^{u,c}| e^{i\delta_{SD}^{u,c}} + i A_{LD}^{u,c}$ , we have

$$|r_f| = \frac{|\lambda^u|}{|\lambda^c|} \frac{\sqrt{|A_{SD}^u \cos \delta_{SD}^u|^2 + (|A_{SD}^u| \sin \delta_{SD}^u + A_{LD}^u)^2}}{\sqrt{|A_{SD}^c \cos \delta_{SD}^c|^2 + (|A_{SD}^c| \sin \delta_{SD}^c + A_{LD}^c)^2}},$$

$$\delta_f = \tan^{-1}(\tan \delta_{SD}^u + \frac{A_{LD}^u}{|A_{SD}^u| \cos \delta_{SD}^u}) - \tan^{-1}(\tan \delta_{SD}^c + \frac{A_{LD}^c}{|A_{SD}^c| \cos \delta_{SD}^c}), \quad (3.22)$$

where the reality of  $A_{LD}$  has been used, and  $\mathcal{A}_f$  and  $\Delta S_f$  can be obtained by using Eq. (1.8). It is interesting to note that in the absence of LD contributions we have  $|\delta_f^{SD}| = |\delta_{SD}^u - \delta_{SD}^c| \lesssim \pi/4$  for some typical SD (perturbative) strong phases and, consequently, we expect  $|\Delta S_f/\mathcal{A}_f| \gtrsim 1$ . This result generally does not hold in the presence of FSI. For example, in the case of  $|A_{LD}^u| \ll |A_{SD}^{u,c}| \lesssim |A_{LD}^c|$ , it is possible to have  $|\Delta S_f/\mathcal{A}_f| \lesssim 1$ . From Table V we obtain

$$\begin{aligned} \Delta S_{\phi K_S}/\mathcal{A}_{\phi K_S} &\approx -1.3 (1.5), & \Delta S_{\omega K_S}/\mathcal{A}_{\omega K_S} &\approx -0.08 (-1.7), & \Delta S_{\rho^0 K_S}/\mathcal{A}_{\rho^0 K_S} &\approx 0.08 (-1.1), \\ \Delta S_{\eta' K_S}/\mathcal{A}_{\eta' K_S} &\approx -0.05 (0.6), & \Delta S_{\eta K_S}/\mathcal{A}_{\eta K_S} &\approx -2.0 (-1.1), & \Delta S_{\pi^0 K_S}/\mathcal{A}_{\pi^0 K_S} &\approx 1.2 (-1.8), \end{aligned} \quad (3.23)$$

where the SD contributions are shown in parentheses. For the  $f_0 K_S$  mode,  $\Delta S/\mathcal{A} \approx 3.0$  but there is no reliable estimate of the FSI effect. From Eq. (3.23) we see that the relation  $|\Delta S_f/\mathcal{A}_f| \gtrsim 1$  indeed holds at short distances except for the  $\eta' K_S$  mode where  $|\delta_f^{SD}| \sim 65^\circ$ . The sign flip of  $\Delta S_f/\mathcal{A}_f$  in the presence of LD rescattering is due to the sign switching of either  $\mathcal{A}_f$  (for  $\phi K_S$  and  $\pi^0 K_S$ ) or  $\Delta S_f$  (for  $\rho^0 K_S$ ).

#### IV. CONCLUSIONS AND COMMENTS

In the present work we have studied final-state rescattering effects on the decay rates and  $CP$  violation in the penguin-dominated decays  $B^0 \rightarrow (\phi, \omega, \rho^0, \eta', \eta, \pi^0, f_0) K_S$ . Our main goal is to understand to what extent indications of possibly large deviations of the mixing-induced  $CP$  violation seen in above modes from  $\sin 2\beta$  determined from  $B \rightarrow J/\psi K_S$  can be accounted for by final-state interactions. Our main results are as follows:

1. We have applied the QCD factorization approach to study the short-distance contributions to the above-mentioned seven modes. There are consistently 2 to 3  $\sigma$  deviations between the central values of the QCDF predictions and the experimental data.
2. The differences between the  $CP$  asymmetry  $S_f^{SD}$  induced at short distances and the measured  $S_{J/\psi K_S}$  are summarized in Eq. (2.18). The deviation of  $S_f^{SD}$  in the  $\omega K_S$  and  $\rho^0 K_S$  modes from  $\sin 2\beta$  is a 2 to 3  $\sigma$  effect owing to a large tree pollution. In contrast, tree pollution in  $\eta' K_S$  is diluted by the QCD anomaly and/or final-state rescattering both of which carry negligible  $CP$ -odd phase. The long-distance effects on  $S_f$  are generally negligible except for the  $\omega K_S$  and  $\rho^0 K_S$  modes where  $S_f$  is lowered by around 15% for the former and enhanced by the same percentage for the latter and  $\Delta S_{\omega K_S, \rho^0 K_S}^{SD+LD}$  become consistent with zero within errors.
3. Final state rescattering effects from charm intermediate states can account for the discrepancy between theory and experiment for the branching ratios of the modes  $\omega K_S, \eta' K_S, \phi K_S$ , and  $\pi^0 K_S$ . Moreover, direct  $CP$  asymmetries in these modes are significantly affected; the signs of  $\mathcal{A}_f$  in the last two modes are flipped by final-state interactions. Direct  $CP$  asymmetries in the  $\omega K_S$  and  $\rho^0 K_S$  channels are predicted to be  $\mathcal{A}_{\omega K_S} \approx -0.13$  and  $\mathcal{A}_{\rho^0 K_S} \approx 0.47$ , respectively, which should be tested experimentally.

4. For the  $f_0(980)K_S$  mode, the short-distance contribution gives  $\Delta S/\mathcal{A} \approx 3.0$ , but at present we cannot make reliable estimates of FSI effects on this channel.
5. Direct  $CP$  asymmetry in all the  $(b \rightarrow s)$  penguin-dominated modes is rather small ( $\lesssim$  a few %) except for  $\omega K_S$  and  $\rho^0 K_S$ . This strengthens the general expectation that experimental search for direct  $CP$  violation in  $b \rightarrow s$  modes may be a good way to look for possible effects of new  $CP$ -odd phases (see e.g. [36]).
6. Since the mixing induced  $CP$  parameter  $S_f$  (actually  $\Delta S_f \equiv -\eta_f S_f - S_{J/\psi K_S}$ ) and the direct  $CP$  parameter  $\mathcal{A}_f$  are closely related, so are their theoretical uncertainties. Based on this study, it seems rather difficult to accommodate  $|\Delta S_f| > 0.10$  within the SM, at least in the modes we study in this paper (except for  $f_0 K_S$  which we cannot make reliable estimates).
7. In particular,  $\eta' K_S$  and (to some degree)  $\phi K_S$  appear theoretically cleanest in our picture; i.e. for these modes the central value of  $\Delta S_f$  as well as the uncertainties on it are rather small. This also seems to be the case in QCDF [39]. Note also that the experimental errors on  $\eta' K_S$  are the smallest (see Table III) and its branching ratio is the largest, making it especially suitable for faster experimental progress in the near future [40].
8. The sign of  $\Delta S_f$  at short distances is found to be positive except for the channel  $\rho^0 K_S$ . After including final-state rescattering effects, the central values of  $\Delta S_f$  are positive for all the modes under consideration, but they tend to be rather small compared to the stated uncertainties so that it is difficult to make reliable statements on the sign at present. However, since the  $S_f$  and  $\mathcal{A}_f$  are strongly correlated, improved measurements could provide enough useful information that stronger statements on the sign could be made in the future.

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